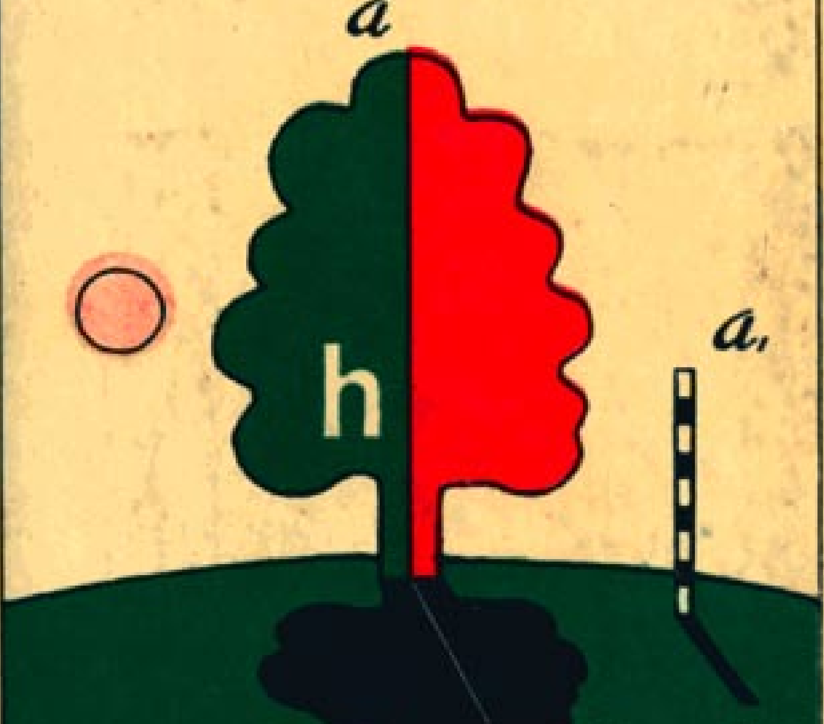
Physical Problems for Robinsons | 116?., 116!.

V. Lange



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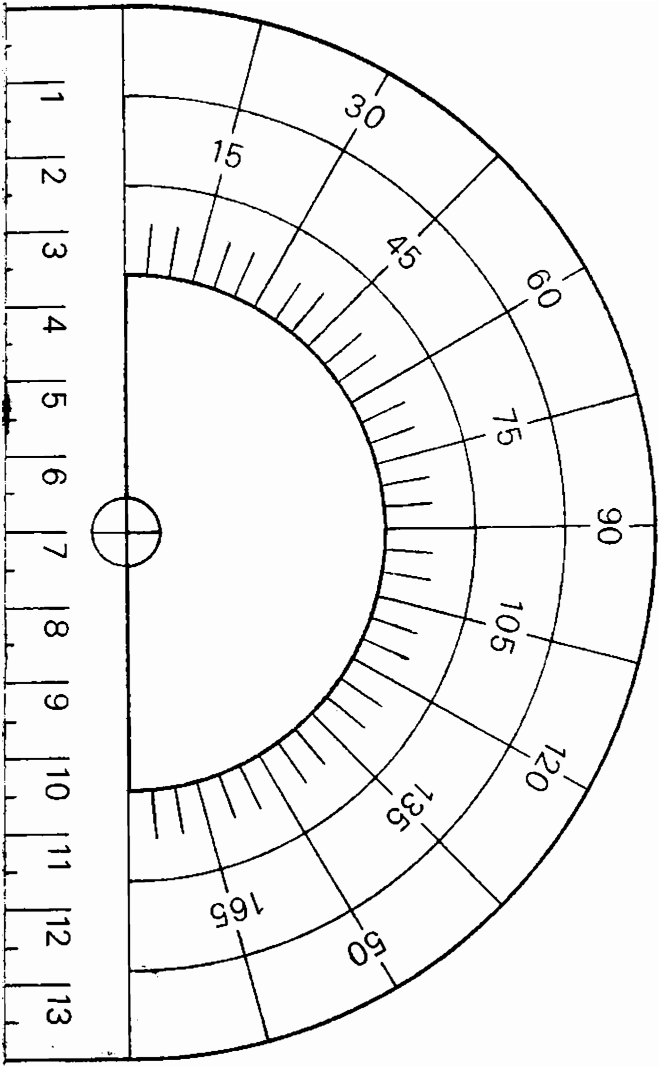

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ИЗДАТЕЛЬСТВО • МИР •

В. Н. Ланге

ЭКСПЕРИМЕНТАЛЬНЫЕ ФИЗИЧЕСКИЕ ЗАДАЧИ НА СМЕКАЛКУ

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Physical Problems for Robinsons

by V. Lange

Translated from the Russian
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A collection of 116 physics experiments and problems for "O" level physics and general science classes, school quizes, and home experimentation. Differs from the usual school collection of problems in being classified by place or occasion of performance rather than by the field of physics. Provides hints on how to go about the experiments and full explanations of the solution.

На английском языке

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FOREWORD

Man constantly has to meet with the necessity of measuring some quantity in all domains of his activity, science or technology, industry or agriculture, space travel or medicine. The quantity may be the temperature of the air or the height of a mountain, the volume of a body or the age of an archaeological discovery. In some cases the necessary measurements can be made with instruments or tools suited for the particular purpose. For example, linear dimensions of a body are measured with rulers, tape-lines, micrometers, vernier callipers, temperatures are measured with thermometers, mass by using balances. Such measurements are called direct. However, in most cases instead of performing a direct determination of the quantity in question one has to measure other quantities and then to calculate the sought one from appropriate formulas. Such measurements are called indirect. Thus, to determine the density of a substance we usually measure the mass and the volume of a body consisting of this substance and then divide the first value obtained by the second one.

As a rule specially developed "standard" methods of measuring quantities are available; examples of such methods (the measuring of a length and of density) have been just cited. There are cases, however, when the usual method of measurement proves unsuitable or even impossible.

Suppose it is necessary to determine the diameter of a thin capillary, in a common thermometer, say, for medical use. The inner diameter is so small that it is quite impossible to insert a ruler or any other tool. Besides, the ruler which you have at hand is too coarse an instrument for such a measurement. What is to be done? It proves expedient to use an indirect method of measuring the diameter of the capillary, rather than a direct one; it is possible to apply not one but many different methods. You will know one of them after studying the solution of problem No. 82 in this book. The standard method of measuring density is not always applicable either. Indeed, we shall encounter difficulties from the start if we decide to determine the mean density of the matter of a planet: you cannot put it on the scale! One has to devise devious ways two of which are described in the solutions of problems Nos. 108 and 112.

In some of the problems given in this book we offer the use of what seem to be utterly unsuitable instruments and objects in order to determine various quantities. The problems, however, can be solved if these objects are applied with due skill. If the solution of the problem proves to be difficult look into the section entitled "Hints and Pointers", and if this does not help either look into the "Solutions", the last part of the book. In any case check your answers even

if you solve the problem without assistance. Possibly your solution proves simpler and more elegant. If so your pleasure will be enhanced.

There are also problems in this book that do not require a quantitative determination of any observables, only a method of performing a certain operation should be proposed. They have in common with problems of the type mentioned above a certain peculiarity, either of the initial situation or of the set of objects which are permitted to be used. (It should be said, to be exact, that some of these problems only seem peculiar. For example, the mean density of the matter constituting the Earth was determined just in the way described in the solution of problem No. 112.)

Though all the problems are of an experimental nature it is only important to *point out* the way to solving, which is correct in principle. It is implied that the instruments and tools mentioned in the text of the problem are ideally precise and that the use of materials "at hand" is allowed, if nothing is specified to the contrary. For example, it is taken for granted that you can always find at home a glass or a reel of thread, some water from the tap or the well, etc.

The quantity required can be found in some cases only approximately, using the available set of objects, but sometimes even estimated results are of great value. Thus, for example, at present the wave-

length of light can be measured with a striking precision (remember that 1 m in SI is defined as a segment which can comprise, in a vacuum, 1650763.73 wavelengths of the gas krypton spectrum, or more exactly of its isotope whose atomic mass is 86). However, rather rough experiments of the English physicist T. Young, made in 1802, in which he determined the wavelength of light for the first time, were of a great value, since even the order of magnitude had not been known.

Most of the situations offered by the conditions of the problems look artificial and hardly can really occur, but all that can happen is beyond foreseeing and one must be ready to meet with the unexpected. Knowledge of the history of science shows that experimenters are often forced to use "stratagems", to devise various indirect and complex methods of measuring quantities and investigating phenomena. Suffice it to recollect that the determinations of the charge of an electron, of the chemical composition of stars, of the structure of the atomic nucleus and very many other things have been achieved just by indirect methods. In all these cases scientists were assisted by imagination, this quality being of great value to those who work in natural science. The reader will also profit by checking his inventiveness in simple cases, so as not to be taken aback in more complicated ones.

This book has little likeness to a school

collection of problems, since in solving many of them items of knowledge from many different divisions of physics may be required. Therefore the author decided to divide the problems into groups according to the situation in which the task is to be performed. In each group the problems are arranged in the order of increasing difficulty, though certainly the author's opinion on the complexity of the problem need not necessarily coincide with the reader's.

Most of the problems in this book have been composed by the author and are published here for the first time. However, a certain number have been drawn from published collections.

The book is intended, in the first place, for pupils of "O" level classes, who have already mastered a considerable amount of knowledge in physics, but there are many problems which pupils who only have started to study this wonderful science will be able to cope with. It seems that the book can prove useful to teachers of secondary school, for example, in compiling a questionnaire for a school party devoted to "entertaining physics".

It is recommended to use a reference book of physical quantities when solving some of the problems. Most of the necessary data can be found in appendices to collections of school problems in physics, but occasionally the reader will have to look into more detailed tables that are published in handbooks.

The author

PROBLEMS

At Home

1. You are to determine the density of sugar. How can you do it if you have at your disposal only a graduate for domestic use and the object of the experiment is castor sugar?

2. Given a 100 g weight and a graduated ruler. How can you determine approximately, with their use, the mass of a certain body if its mass is not greatly different from that of the weight? How will you proceed if instead of the weight you are given a set of coins?

3. How can you determine the capacity (volume) of a pan using a balance and a set of weights?

4. A cylindrical glass is filled with a liquid to the brim. How can you divide the contents of the glass in two strictly equal parts if you have one more vessel which is somewhat smaller and of another shape?

5. Two friends were taking a rest on a balcony. Having nothing to do they pondered on how could one determine whose box of matches had less matches left, without opening the boxes. What method do you suggest?

6. In what way can the centre of gravity of a smooth stick be determined without using any tool?

7. How would you determine the diameter of a foot-ball with a *stiff* (wooden, say) ruler?

8. How can the diameter of a small ball be found using a graduate?

9. The diameter of a comparatively thin wire must be determined as precisely as possible having for this purpose a "checked" copy-book and a pencil. How would you do it?

10. A vessel of rectangular shape is full of water. A body floats in it. How can one determine the mass of the body using only a ruler?

11. Given a steel knitting needle and a graduate full of water. How can you determine the density of a piece of cork?

12. Describe the determination of the density of the wood from which a small stick is made if it floats in a narrow cylindrical vessel and only a ruler is at your disposal.

13. A glass stopper has a cavity inside it. You are to determine the volume of the cavity without breaking the stopper. Can it be done using a balance, a set of weights and a vessel full of water?

14. A steel sheet is nailed to the floor. Given a light wooden cylindrical rod (a stick) and a ruler. Devise a method of determining the coefficient of friction between wood and steel using only the objects named.

15. You are in a room lighted by an electric lamp. Which of the two converging lenses you have has greater power? No special instruments are available for your purpose. Point out the way to the solution of the problem.

16. Given two lenses: a converging lens and a diverging one. Which of them has greater lens power? How can you determine it without resorting to any instrument?

17. A long passage in which there are no windows is lighted by an electric lamp. The lamp can be switched on and off with a switch installed at the entrance. This is inconvenient for a person going out into the street, since he has to reach the exit in the dark. However, the person entering the passage and

switching the lamp on is also dissatisfied with the arrangement: having walked through the passage he leaves the lamp burning uselessly. Is it possible to devise a circuit permitting the lamp to be switched on and off from any end of the passage?

18. Suppose you have been required to use an empty can and a stop-watch in order to measure the height of a house. Are you able to cope with the task? Describe how you would do it.

19. How can one determine the outflow velocity of water from a tap having a cylindrical jar, a stop-watch and a vernier calliper?

20. Suppose you have to fill with water a large tank of known capacity using a flexible hose with a cylindrical nozzle. You want to know how much time this tedious task will take. Could you perform the calculation with only a ruler at your disposal?

21. The pressure inside a foot-ball is to be determined with a sensitive balance and a ruler. How is it to be done?

22. Given a burnt-out electric bulb. How can one determine the pressure inside the bulb using a cylindrical vessel of water and a graduated ruler?

23. Try to solve the preceding problem when you are allowed to use a pan full of water and a balance with a set of weights.

24. Given a narrow glass tube sealed at one end. The tube contains some air separated from the ambient atmosphere by mercury. Given also a ruler graduated in millimetres. Determine the atmospheric pressure.

25. How can the latent heat of steam be determined using a refrigerator, a pan of unknown capa-

city, a watch and a steadily burning gas-stove burner? Consider the specific heat of water to be known.

26. It snows outside but the room is warm. Unfortunately there is no way of measuring the temperature—no thermometer. But there is a storage battery, a precision voltmeter, also a precision ammeter, copper wire galore and a detailed reference book of physics. Can one find the temperature of the room air using the above objects?

27. How can one solve the preceding problem if there is no reference book, but it is permitted to use, besides the objects enumerated, an electric stove and a pan of water?

28. The daughter asked her father who was writing down the reading of the electricity meter to let her go for a walk. Giving the permission the father asked his daughter to be back just in an hour. How can the father check the duration of the daughter's walk without using his watch?

29. Problem No. 17 is published often enough in various collections and therefore is well known. Here is a like problem but somewhat more complicated. Devise a circuit which makes it possible to switch on and off an electric lamp or any other instrument connected to the mains from *any number* of locations.

30. If you put a small wooden cube on the cloth covering the disc of the record-player of your radio-set close to the axis of rotation the cube will rotate with the disc. When the distance from the axis is great the cube, as a rule, is thrown off the disc. How can one determine the coefficient of friction between wood and cloth using only a ruler?

31. Work out a method of determining the volume of a room if you have at your disposal a suf-

ficiently long thin thread, a watch and a small weight.

32. In teaching music, choreography, in training sportsmen and for some other purposes a metronome is often used. This is an instrument marking time by periodical abrupt clicks. The interval between two clicks is controlled by displacing a small weight sliding on a special oscillating scale. How can one graduate the scale of a metronome in seconds, if this has not been done at the factory, using a thread, a steel ball and a tape-line?

33. The small weight of a metronome whose scale has not been graduated (see the preceding problem) is to be placed at a point such that the interval between two clicks be equal to one second. You are allowed to use a long ladder, a piece of brick and a tape-line for this purpose. How would you make use of this set to complete the task?

34. One of the dimensions of a rectangular parallelepiped made of wood is considerably greater than the two others.

How can one determine the coefficient of friction between this bar and the surface of the floor in a room using only a ruler?

35. Two hollow spheres of equal weight and volume are covered with the same paint and it is undesirable to scratch the surface. One of the spheres is made of aluminium, the other of copper. By what simple method can one ascertain which of them is of aluminium and which of cooper?

36. How can one obtain the weight of a certain body using a strong homogeneous graduated lath and a piece of not very thick copper wire? It is permitted to use a handbook of physics.

37. It is required to determine the radius of a spherical mirror (or the radius of curvature of a *concave* lens) using a stop-watch and a steel ball of known radius. How can it be done?

On a Ramble

38. A grown-up and a child want to cross a brook; one from the left bank onto the right, the other in the opposite direction. There are two planks, one on each bank, but each plank is somewhat shorter than the distance between the banks. How can the grown-up and the child manage to cross the brook?

39. How can the length of a lightning be estimated in some cases from the duration of the following thunder having only a stop-watch at one's disposal?

40. A bell is suspended from a pole. The bell is given blows at regular intervals of one second precisely. Can one by observing the blows and listening to the sounds of the bell determine the velocity of sound propagation taking measurements only with a tape-line?

41. How can one, using a ruler, find on a sunny day the height of a tree without climbing it?

42. In some towns of the USSR electronic devices have been installed that automatically calculate and show on a screen the speed which the car drivers should keep in order to arrive at the next traffic lights when the signal is green. Usually the sequence of the changing signals on the screen is as follows: first 45 km/h, then 50 km/h, 55 km/h and finally 60 km/h; after this the signals on the screen go out, since a speed over 60 km/h is permitted only on a limited number of streets. How can one, standing at the

crossing and observing the signals on the screen, calculate the distance to the next traffic lights using only a stop-watch?

43. Two boys on a skating-rink want to compare their respective masses, whose mass is greater and how many times. How can they fulfil the comparison using only a tape-line?

44. You are standing on the bank of a small river in the evening; there is a lantern post on the other bank. How can you determine the distance to the post and its height if you are to use only a short wooden lath and a tape-line?

45. Determine the initial velocity of a bullet fired from a toy-pistol using only a tape-line.

46. How can one solve the preceding problem if instead of a tape-line the experimenter is to use a stop-watch?

47. How can one, using a tape-line, determine how many times greater is the initial velocity of a ball thrown by a boy than that of a ball thrown by a girl?

48. You want to determine the breadth of a river in paces. How can you do it, of course approximately, using a blade of grass picked on the river bank?

49. To determine the direction of the magnetic meridian you are permitted to use a glass of water, a pinch of sal ammoniac (NH_4Cl), a pair of scissors, a coil of copper wire, a small zinc plate and a cork. How can you perform the task with the use of these objects?

50. Suppose you have been provided with a saucer of mercury, a protractor, a small weight and a thread and you are required to determine the height

of a tower (or any other building). Will you be able to do it knowing the dimensions of the parts of your own body?

51. How can one determine the height of a mountain using a heating appliance, a pan of water and a precision thermometer?

52. You are to measure the candle power of an inaccessible source of light. In order to do it you may use an instrument for measuring the intensity of illumination (luxmeter) and a tape-line. Describe the necessary experiment.

53. Suppose you are on a rotating platform (like a side-show in a recreation park). The platform is fully enclosed so that the surrounding objects are invisible. You want to determine the direction of rotation of the platform. How can you solve the problem using a small steel ball?

On the Lake

54. Without using any instrument show that the coefficient of surface tension of a soap solution is less than that of pure water.

55. There was no wind, the weather was calm, two friends went for a row on a lake in two boats absolutely identical as to the shape and dimensions. During the excursion they decided to have a race with each other. Wishing the race to be absolutely fair the boys decided to distribute the load they had so that the weights of the two boats be equal. How could they implement their plan using a long rope they had?

56. A man in a boat wants to find its mass. Can he do it knowing his own mass but having only a long rope at his disposal?

57. Having crossed the lake from the shore where their camp was the tourists looked at the watch and decided it was time to take a short rest. The weather was calm and the radio news from the camp loudspeaker could be heard when their portable set was switched off. One of the tourists then said that the distance from the camp was about three kilometres. How did he find the distance?

58. A skin-diver had to determine the depth of a lake. Unfortunately he had only one instrument—a cylindrical graduate. However, the skin-diver managed to perform the task. Could you say how he achieved it? Can the problem be solved if a conical graduate is substituted for the cylindrical one?

59. Buying a nylon fishing-line at a shop an angler forgot to ask what load the line could bear. After some reflection he devised a method of determining the value of the load using a weight of 1 kg and a protractor which he happened to have with him. Try to guess how the angler solved the problem.

60. Will the angler be able to determine the strength of the fishing-line if he has a weight of 1 kg and a tape-line?

61. An angler decided to calculate the ultimate strength (i.e. the ratio of the breaking force to the area of the cross section of the line, called also the tensile strength) of the material of the line, having a piece of fishing-line of known length and diameter, a weight and a stop-watch. How is the experiment to be conducted in order to obtain the value in question?

62. A stone has been thrown into calm lake water. How can one determine approximately how far the stone was thrown using a one-metre ruler and a stop-watch?

During a Journey

63. How can you determine in calm weather the velocity of falling raindrops by observing the streaks left by the drops on the window-panes of a moving railway car? To solve the problem it is permitted to use only a stop-watch and a protractor.

64. How can one, using a graduated scale, determine the velocity of falling raindrops by observing the marks they leave on the side-windows of a moving car? The weather is calm.

65. When starting from a station the train moves for some time practically with a uniform acceleration. Devise a method of determining the acceleration during this period using a thread, a 100-g weight and a graduated scale.

66. How will you solve the preceding problem if instead of a ruler you are given a dynamometer, a precise spring-balance?

67. How could problem No. 65 be solved if the experimenter had been offered a protractor instead of a graduated ruler?

68. A thermometer which measures the temperature of the air outside and a precise counter of the revolutions of the wheel are installed in a driving car of a suburban electric train. How can one measure the termal coefficient of expansion of the metal of which the car wheels are made?

69. Suppose you are driving a car on a level stretch of the motor road. How can you determine approximately the coefficient of resistance to the car motion using only the instruments on the car instrument board?

70. How can one determine the slope of the motor road (the angle between the road surface and

the horizontal) using a wooden bar and a dynamometer?

71. How can you find the polarity marks of a car storage battery using a driver's portable lamp, a piece of wire and a compass?

72. How can the preceding problem be solved if you dispose of two leads and a glass of water?

73. How can problem No. 71 be solved if you have two copper wires and a raw potato?

74. A man driving a car was asked to determine the slope of the road. In order to perform the task he was provided with a hoop and a stop-watch. What is he to do?

In the School Lab

75. You have two pendulums. The period of oscillation of one of them is known. What is the simplest method of finding the period of the other one?

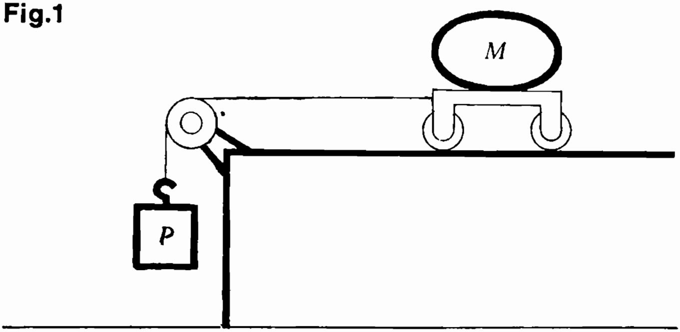
76. You are asked to select from several kinds of filter paper the one whose pores are smaller. How can you do it using no instruments?

77. Two metal bars looking absolutely identical are lying in a table drawer. One of them is made of soft iron, the other is made of steel and is magnetized. How can you, using only the two bars, tell which is the magnet?

78. One of two absolutely identical spherical glass flasks is filled with water, the other with spirit. The two flasks are tightly corked up. How can you, using a table lamp, determine without uncorking the flasks which of them is filled with water and which contains spirit?

79. It is required to find the value of mass M using weights, a stop-watch and the installation shown in Fig. 1. What is the simplest method of doing this?

Fig.1



80. Two coils are placed on a closed iron core. How can you determine the number of turns in each if you have at your disposal a source of alternating current, a coil of insulated wire and a highly sensitive multirange voltmeter?

81. You are to determine the mass and length of a copper wire of which the coil of an electromagnet is wound. It is undesirable to unwind the coil. Can the problem be solved if you have a current supply, a voltmeter, an ammeter and a micrometer?

82. How can one determine the inside diameter of a uniform glass capillary, for example of a medical thermometer, using a ruler, a small rubber syringe, a precise balance with a set of weights and a drop of mercury? The ruler is too coarse an instrument to be used for measuring the diameter directly.

83. In order to measure the velocity of a rifle bullet the experimenter can dispose of an electric motor whose speed of rotation is known, two cardboard discs, some glue, a ruler and a protractor. What use should he make of this set of objects?

84. You are required to find the weight of a body using a support, a spring, a ruler and an only weight of known value. How can you do it?

85. Given a wooden plank, a wooden bar and a ruler. Work out a method of determining the coefficient of friction of wood against wood using only the objects enumerated.

86. How can one determine the coefficient of friction of a bar against an inclined plane on which the bar is placed? The slope of the plane is constant and not too great so that unless an external force is applied the bar will not glide.

87. Usually, to determine the mass of a body we use a balance and a set of weights. Now, what can we do if no balance is available? One method of "weighing" a body without a balance was discussed in problem No. 79. A new variant should be worked out since we have now only one weight, and instead of the installation shown in Fig. 1 we are permitted to use a coil of thin but strong cord, a light pulley-block, and a stop-watch.

88. How can you find the mass of a small steel bar using a spirit-lamp, a jar of water, a calorimeter, a thermometer and a graduate? You are permitted to refer to a handbook of physics. The mass of the calorimeter and the material it is made of are known.

89. How can one determine the approximate temperature of a highly heated steel bar using the instruments enumerated in the condition of the pre-

ceding problem if the thermometer is designed for measuring temperatures not over 100 °C?

90. Given a storage battery, whose electromotive force and inner resistance are not known, an ammeter, connecting leads and two resistances, only one of them known. How can the value of the unknown resistance be determined?

91. You have at your disposal a Grenet cell (the electrodes are zinc and carbon, the electrolyte—diluted sulphuric acid with an addition of potassium dichromate as depolarizer), a precise balance with a set of weights, a rheostat, an ammeter, a ruler, a handbook of physics and a map of the district. How can you determine, using the given set of objects, the average speed at which your friend covers on a bicycle the distance from your town to a neighbouring settlement and back?

92. Given an intricate electric circuit consisting of identical capacitors of known capacity C, connected in series and in parallel. The capacity of the storage battery was calculated theoretically. Since the circuit is very complicated it is natural to try and check the theoretical value experimentally. For this purpose you are offered a set of equal resistances, a voltmeter, an ammeter and a storage battery. How will you proceed?

93. How can one measure the volume of a lecture-room if for this purpose one disposes of a coil of copper wire, a balance with a set of weights, a storage battery, a voltmeter, an ammeter and a handbook of physics?

94. Devise a method of measuring the volume of the lecture-room if you are permitted to use, out of the set of objects listed in the preceding problem,

only the coil of wire and the balance with the set of weights.

95. A tuning fork made of invar (an alloy with a negligible coefficient of expansion) has a frequency of 440 Hz practically independent of temperature. How can one determine the temperature in the laboratory by counting the number of beats between the oscillations of the tuning fork and of the organ pipe producing at 0 °C also 440 oscillations per second, if the length of the pipe is not altered with temperature?

At the Factory

96. How can you ascertain whether a hack-saw blade has been magnetized or not without using any instruments or other objects?

97. Suggest a method of determining the area of a homogeneous plate of irregular shape using an angle-rule with graduation marks, scissors and a balance with a set of weights.

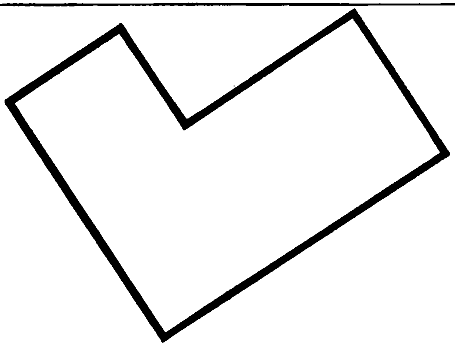
98. Given a vessel with a melted substance and a bit of the same substance in a solid state. How can one predict, without waiting for solidification to take place, what changes will occur in the volume of the melted substance with transition into the solid state?

99. How can one, using only a ruler which has no graduation marks, determine the location of the centre of gravity of a homogeneous metallic plate (Fig. 2) whose all angles are right ones?

100. By what method can one, using a strong magnet (preferably horseshoe-shaped), determine what kind of current, alternating or direct, is supplied to the electric lamp?

101. A wattmeter (an instrument designed to measure the power consumed by electric installations from the mains) has two pairs of terminals to which the two coils of the wattmeter are connected: the current coil connected in series with the installation and having consequently a small resistance and the voltage

Fig. 2



coil connected in parallel. The designations at the terminals are obliterated. Can one, without opening the instrument and using two leads and an electric pocket-light, determine what coils are connected to what terminals?

102. How can one determine with a voltmeter at what end of the two-wire transmission line the source of the current is situated?

103. An unskilled turner made a batch of parts with a wrong dimension; as a result each part weighs 10 g less than it should. Before remelting, the parts with defects were stored in a separate box at the factory store and the box was placed alongside

nine other similar boxes of the same parts but correctly made, which had accordingly the required weight.

The absent-minded storekeeper forgot which box contained the rejected parts. This can be easily ascertained by weighing one after another the parts from all the boxes. But the parts to be remelted may happen to be in the last box, so that nine weighings may have to be performed (the weighing of the part from the tenth box can be dispensed with, since if the parts from the first nine boxes had no defects then the rejected ones must be in the last box). However, the store head said that to find the required box one weighing would be enough. What is the storekeeper to do?

104. How can one inspect the surface of a part fixed in the lathe chuck without stopping the machine tool?

105. A commutator a-c motor with series excitation is connected to the mains through a rheostat which makes it possible to change the motor speed smoothly. How can you determine the motor speed of (a) 750 rpm, (b) 1500 rpm if you have at your disposal a neon lamp, a pair of compasses, a pencil, a ruler, some glue, scissors and a sheet of cardboard?

In Outer Space

106. An astronaut is in open space and he wants to return to the spaceship. On the Earth there is no problem, you have just to start walking but in space everything is far more complicated, you have nothing under your feet to push off. How is the astronaut to proceed in order to start moving?

107. In order to find the mass of a body we use either a balance with scales or a spring balance. It seems that neither of them can work in the state of weightlessness, for instance on a small artificial satellite of the Earth or on board a spaceship which is moving with her engines shut off. What would you do if you were required to find the mass of a body under such conditions using just a balance? What should you use: a balance with scales or a snring balance and how?

108. Having reached an unknown planet tne spaceship shut off her engines and started to move in a circular orbit; the astronauts began their preliminary investigations. Are they able to determine the mean density of the matter of the planet using only their watches?

109. In order to determine experimentally the gravity acceleration on a newly discovered planet the astronauts decided to make use of a small steel ball, a powerful lamp, an electric motor of constant (known) speed, on whose axle a cardboard disc is fixed which has a narrow radial slit, a piece of black linen, a graduated ruler and a camera with a highly sensitive film. How should they use this set of objects in order to solve the problem?

110. How can the preceding problem be solved using a weight of known mass and a spring balance (dynamometer)?

111. Astronauts decided to determine the mass of the planet onto which they had been brought by a rocket ship. For this purpose they used a spring balance and a 1-kg weight. How did they implement their decision if the planet radius was known to them from previous astronomic measurements?

112. Continuing to study the planet on which they had alighted the astronauts engaged in a second determination (see problem No. 108) of the mean density of the planet matter. Indicate how they should do it if they have at their disposal a thin thread of known length, a small weight and a stop-watch. The astronauts also know the length of the equator.

113. Observing on his TV-set the landing of astronauts on the surface of the Moon a teacher of an American college noticed near one of the ship compartments close to the figure of an astronaut a heavy object swinging on what looked like a rope. Having looked at his watch the teacher managed to determine the acceleration of gravity on the Moon. How did he do it?

114. How can the astronauts, on arriving at an unknown planet, determine, using a sensitive galvanometer and a coil of wire, whether the planet has a magnetic field?

115. An astronaut who got out into open space and is in no manner connected to the ship wants to turn by 180°. What should he do?

116. Imagine that life originated on Venus and in the course of time creatures endowed with reason appeared ("humanoids", a term used by modern sci-fi writers). In advancing their science these humanoids, owing to specific surroundings, would constantly meet with difficulties quite unknown to inhabitants of the Earth. The clouds on Venus, for instance, are so dense that the inhabitants of the planet would never be able to see the heavenly bodies. The question arises: would they be able to guess that Venus rotates about its axis and determine the direction of rotation? Try to suggest your own method.

HINTS AND POINTERS

1. The domestic graduate has on it several different scales for dry goods: flour, manna-croup, castor sugar, etc. These scales are in grams. There is also one scale for any liquids in cubic cm.

2. Support the ruler at its midpoint putting it, for instance, on the edge of a triangular file and recall the condition of equilibrium of a lever.

3. Determine the weight of the empty pan and then do it with the pan filled with water.

4. Think of the way to draw a plane that will divide a cylinder into two parts of equal volume.

5. First think of the cause which makes the velocity of a falling parachutist drop abruptly with the opening of the parachute.

6. The stick will be in equilibrium if it is supported at its centre of gravity.

7. A sphere rolling on a plane covers in one turn a distance equal to the length of its circumference.

8. The diameter of the ball can be simply expressed in terms of its volume.

9. Wind several turns of the wire closely on the pencil.

10. Recall Archimedes' law.

11. In solving this problem one should again apply Archimedes' law. The knitting needle is required to submerge the cork in the water.

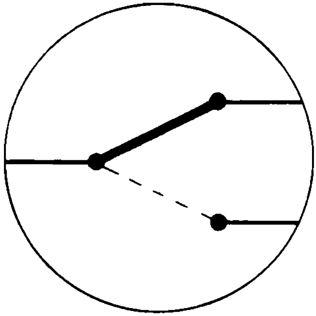
12. The greater the density of the floating body the smaller is the part of its volume over the surface.

13. There is a legend that Archimedes discovered his law while considering how he could verify whether the court jeweller had made king Hiero's

crown of pure gold or substituted silver for a part of it. In our problem the stopper plays the part of the crown, glass that of gold and air that of silver.

14. The stick leaning against the wall begins to slip and falls down if the angle between the stick and the surface of the floor is small enough.

Fig. 3



15. We say that the lens has a greater lens power if it has a shorter focal distance.

16. The lens power of a system of two lenses adjusted closely to each other is equal to the sum of the powers of the individual lenses.

17. The problem can be solved probably in the simplest way by using a single-pole switch (Fig. 3).

18. If you drop the can from the roof of the house you will hear distinctly the sound of the can striking the ground.

19. The larger the diameter of the vessel the slower it fills.

20. The height to which the fountain jet rises is determined by the outflow velocity.

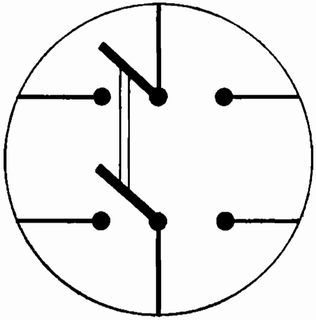
21. The density of a gas depends on the pressure to which it is subjected.

22-24. With constant temperature the volume of the gas is inversely proportional to the pressure.

25. Recall the formulas by which one can calculate the amount of heat necessary to heat and vaporize a liquid.

26, 27. When a metal is being heated its ohmic resistance increases linearly with temperature.

Fig.4



28. The amount of electric energy drawn from the mains is proportional to the power of the appliance connected and to the time of its working.

29. Try to use a two-pole switch (Fig. 4).

30. Consider what force makes the small cube move in a circle, i.e. what is the centripetal force in this case.

31, 32. Recall the formula from which the period of oscillations of a simple (point-mass) pendulum is calculated.

33. The time of falling of a body dropped from a moderate height is easily calculated.

34. Try to communicate motion to the bar by applying force at different heights and observe the behaviour of the bar.

35. It is more difficult to set into rotation a fly-wheel whose mass is concentrated in the rim than a homogeneous disc, if the two wheels are of equal mass.

36. The tensile strength of wire depends on its material and diameter.

37. The centre of a ball rolling on the surface of the mirror has the same motion as a pendulum.

38. If the arms of the lever are of different length a child can balance a grown-up.

39. The farther the source of the sound the later is the sound heard.

40. The velocity of light is 300 000 km/s and that of sound is a little greater than 300 m/s.

41. Make use of the fact that in similar triangles the corresponding sides are all in the same ratio.

42. The motor cars must cover the same distance between the two traffic lights at any speed.

43. Use the second and third Newton's laws.

44. See hint to problem No. 41.

45-47. The greater the velocity of the body thrown at an angle to the horizontal the higher it will rise and the farther it will fall.

48. See hint to problem No. 41.

49. There is a magnetic field around a coil through which an electric current is flowing, similar to that surrounding a permanent magnet.

50. The saucer of mercury should be used as a horizontal mirror in which the reflection of the tower top can be seen.

51. The boiling point depends on the pressure, and pressure decreases with elevation.

52. Make use of the relation between the intensity of illumination and the distance from the source of light.

53. Recall Newton's first law and take into account that a rotating platform is not an inertial reference frame.

54. Observe the behaviour of soap lather thrown on the surface of pure water.

55, 56. Use the basic laws of dynamics.

57. Recall that signals of Greenwich (Moscow) time are usually broadcast before the latest news.

58. Try to apply Boyle's law.

59, 60. Note the fact that whatever the force you apply to the fishing-line with the weight hanging on it you cannot stretch it strictly horizontal.

61. Analyse the expression for the centripetal force.

62. Ripples start to propagate from the point where the stone falls and reach the shore.

63, 64. The velocity of the raindrop with respect to that of the train or motor car is equal to the vector sum of its velocity with respect to the ground and the velocity of the ground with respect to the vehicle. Consider the mutual orientation of the three vectors.

65-67. Consider the forces which act on the weight suspended by the thread.

68. Any body expands when it is being heated.

69. Familiarize yourself with the design and purpose of the car speedometer.

70. Compare the forces necessary for moving the bar up and down the inclined plane.

71. Recall Oersted's experiment and the right-hand screw rule.

72. Immerse the ends of the leads connected to the electrodes into the water and observe the ensuing phenomena.

73. Connect the leads to the electrodes and stick the free ends into a potato half.

74. The time necessary for the hoop to roll down depends on the slope of the road.

75. Set the two pendulums oscillating and observe them.

76. Recall capillarity and the laws which describe it.

77. If you bury a magnet in iron filings, most of them will stick to the poles.

78. The lens power depends on the index of refraction of the material of which it is made.

79. Equal forces give the same acceleration to bodies of equal mass.

80. Recall the design and principle of working of a transformer.

81. The ohmic resistance and the mass of a lead depend on its diameter and length.

82. The mass of a cylinder is directly proportional to its height and to its diameter squared.

83. While the bullet flies between the discs fixed on the axle of the motor, they go on rotating.

84. Use Hooke's law.

85. The bar will not slide on the plane so long as its angle of slope is not great enough.

86. See hint to problem No. 70.

87. Pass the cord over the pulley and bind the load and the weight to its ends.

88, 89. The amount of heat required to change the temperature of a body is proportional to its mass.

90. Make use of Ohm's law for the circuit.

91. The flow of electric current through a galvanic cell is accompanied by the passing of the substance of the negative electrode into solution.

92. In an a-c circuit a capacitor behaves like a resistance, whose value is determined by the capacity of the capacitor and the frequency of the alternating current.

93. The ohmic resistance of a lead is directly proportional to its length.

94. See hint to problem No. 36.

95. In solving this problem one should bear in mind that the velocity of sound is of the same order as the velocity of gas molecules.

96. Break the hack-saw blade into two pieces.

97. The mass of a plate uniform as to its thickness is proportional to its area.

98. A body floats in a liquid if the density of the former is less than that of the latter.

99. The centre of gravity of a system consisting of two bodies is situated on the straight line which connects the centres of gravity of these bodies.

100. In a magnetic field a lead through which a current is flowing is subjected to the action of a force whose direction depends on the direction of the current.

101. The resistances of the two coils of a wattmeter are widely different.

102. The potential difference at the ends of a section is proportional to the current flowing along this section.

103. If two of the three parts taken at random from the three boxes happen to be rejected ones then the overall weight of the two will be 20 grams less than it should be. If, however, among the three parts only one is defective the overall weight will be only 10 grams less than the correct one.

104. Note the fact that after every turn the part will be in the same position.

105. A neon lamp connected to an a-c supply flashes 100 times per second.

106. Recall the principle of action of a rocket.

107. A. Einstein was the first to point out that the inertia forces appearing in systems which are in accelerated motion are equivalent to gravity forces.

108. Write down the formula for the centripetal force in terms of the period of rotation and equate the expression obtained to the gravity force.

109. The distances traversed by a falling body in equal time intervals are related to one another as consecutive odd numbers.

110. Use Newton's second law.

111. Recall the law of gravitation.

112. Read the hints to the two preceding problems.

113. Use the formula for the period of oscillation of a simple pendulum.

114. When the magnetic flux permeating a coil is changed an electromotive force of induction is generated in the coil.

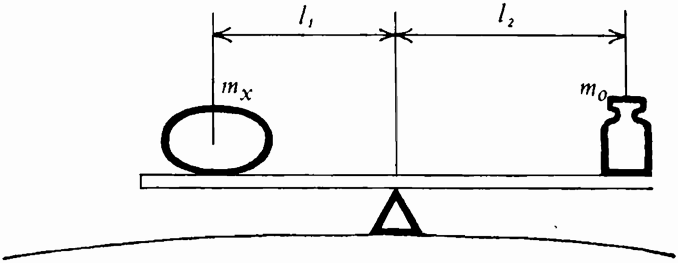
115. If the rotor of an electric motor is fixed the stator begins to rotate, provided there is no obstacle to this rotation.

116. Recall the properties of the gyroscope and pendulum.

SOLUTIONS OF THE PROBLEMS

1. Having poured some castor sugar into the graduate (the make of a domestic graduate was described in "Hints") find its mass using the appropriate scale and its volume from the scale for liquids.

Fig. 5



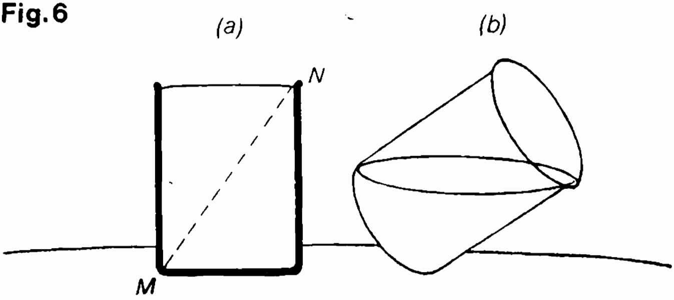
Then calculate the density of sugar in the usual manner. The value obtained will be, however, somewhat less than the true one since the air in the interstices between the sugar granules contributes to the volume found.

2. The solution is illustrated in Fig. 5. Place the midpoint of the ruler on the edge of the file; this is easily done since you can refer to the marks on the ruler. Then adjust the positions of the weight and the body being investigated so that the whole system is in equilibrium. Now from the conditions of equilibrium for a lever we have

$$m_x g l_1 = m_0 g l_2$$

(g—acceleration of gravity) and can find the sought mass m_x. The lengths l_1 and l_2 are read from the ruler scale and m_0 is equal to 100 g as given in the formulation of the problem. In the equality expressing the

Fig. 6



condition of equilibrium of the lever the weight of the ruler can be neglected, since with respect to the axis of rotation the turning moment of the ruler weight is zero.

In order to answer the second question of the problem one must know that the weights of coins in all countries are controlled by law.

3. Let the mass of the empty pan be m_1, and when filled with water m_2. Then the difference $m_2 - m_1$ is the mass of water of a volume equal to the pan capacity. Dividing this difference by the density of water, ρ, we find the volume of the pan

$$V = \frac{m_2 - m_1}{\rho}$$

4. If a plane is mentally drawn through points M and N as shown in Fig. 6a it will cut the cylinder

into two symmetrical parts, which are therefore of equal volume. This fact indicates the solution of the problem. Gradually tilting the glass let the liquid contained in it run off until the bottom just begins to show (Fig. 6*b*). At that moment the liquid left in the glass is just a half of its contents.

5. Two forces act on the falling matchbox: the gravitation of the Earth and the air resistance. The former is determined by the mass and thus is greater for the box which is fuller. At the same time the latter force is equal for the two boxes at equal velocities. Therefore the resultant force will be greater, speaking generally, for the box which is fuller. This box will have a greater acceleration*, and since its velocity increases at a higher rate the box will reach the ground sooner. Thus, the two boxes should be dropped from the balcony simultaneously. That which reaches the ground first contains more matches. Convincing results can be obtained, however, only if the difference in the number of matches is sufficiently large.

6. The simplest way of finding the centre of gravity of the stick is to balance it on the hand, palm vertical. The state of equilibrium, as is well known, witnesses the position of the c.g. over the point of support. However, another more interesting and instructive method of solution is possible.

If you put the stick horizontally on both hands, palms vertical, and slowly move the hands to make the palms touch, they will always meet at the

---

* The acceleration increases with mass, since

$$a = \frac{P - F_{res}}{m} = \frac{mg - F_{res}}{m} = g - \frac{F_{res}}{m}$$

c.g. of the stick and the stick will not fall down in whatever manner the hands come together.

This is due to the fact that when one hand comes nearer to the c.g. the pressure on this hand increases in comparison with that on the other, which is farther from the c.g. Since the friction force increases with pressure this force acting on the hand nearer to the c.g. will surpass the friction force between the stick and the other hand and thus the movement of the stick relatively to the hand nearer to the c.g. will cease and the stick will start to move relatively to the other hand. Thus the centre of gravity will always be between the palms and will be "pinpointed" in the end.

7. It suffices to roll the ball, having wetted it with water, on the floor just one turn and then measure the length of the wet trace. The diameter d is then calculated from the formula

$$d = l/\pi$$

One can also wind a thread, just one turn, on the "equator" of the ball, measure the length of the thread and calculate the diameter as above.

8. First determine the volume V of the ball in the usual way using a graduate and then calculate the diameter d from the formula

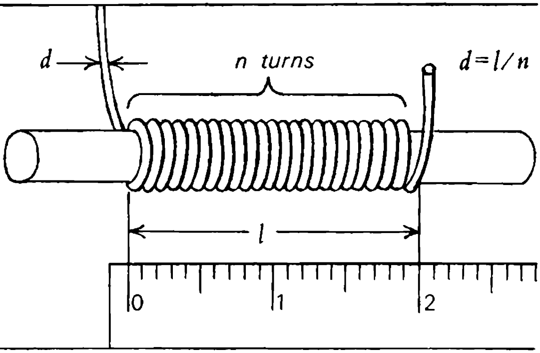
$$d = \sqrt[3]{\frac{6V}{\pi}}$$

9. The side of the square in a school copy-book is half a centimetre long to a high enough accuracy (the author checked several copy-books and in the worst case 40 squares totalled 202 mm instead of the required 200; this corresponds to an accuracy of

1%). This fact can be made use of in solving the problem.

Wind the wire on a pencil keeping the turns close to one another and making the number of turns such that the length of the coil be equal to a whole number of squares. The number of turns should be sufficiently great; otherwise, as will be seen later on,

Fig. 7



the error may be great. Then dividing the length l of the wire coil on the pencil by the number of turns n you will obtain the required value of d:

$$d = \frac{l}{n}$$

(see Fig. 7 on which a ruler with divisions is shown instead of a copy-book page).

The theory of errors shows that the relative error of a fraction is equal to the sum of the relative errors of the numerator and the denominator. In our case l has been determined with a precision of about 1% (see above). In counting the number of turns

the experimenter may make a mistake by one turn, it often being difficult to decide whether the correct number should be taken as 49 or 50 turns. Therefore the relative error of the denominator proves about 0.02. Thus the relative error in determining the diameter of the wire will be 3% and the absolute error (deviation), with a diameter of 0.1 mm, only 0.003 mm, not so bad. Unfortunately, the real error will always be greater as it is difficult to wind the wire turns very close.

10. If a body floats its mass is equal to that of the displaced water. First find the volume of the displaced water multiplying the area of the cross-section of the vessel determined with the ruler by the decrease in the level of the water in the vessel (this is measured also with the ruler) after removing the body. Multiplication of the volume of the displaced water by its density gives the mass of the water and consequently the mass of the floating body.

11. Placing the cork into the graduate and reading the increase in the water level, determine the mass of the stopper by the method described in the solution of the preceding problem. The volume of the piece of cork can be found by submerging it in the water with a knitting needle. After this the density is easily calculated.

12. Besides the method described in the preceding problem another one can be suggested. Measure the overall length of the small stick and put it into the water. Let the total length of the stick be l_1 and that of the submerged part l_2. Then the mass of the stick is

$$m_1 = l_1 S \rho_1$$

where ρ_1 is the density of the wood and S is the area of the cross-section of the stick.

The mass of the displaced water is

$$m_2 = l_2 S \rho_2$$

where ρ_2 is the density of water. Since the stick floats the two masses are equal:

$$l_1 S \rho_1 = l_2 S \rho_2$$

Hence

$$\rho_1 = \rho_2 (l_2/l_1)$$

It is important that the stick float in a vertical position or at least in a slightly inclined one for if it floats "flat" the solution of the problem will prove considerably more difficult. The narrow cylindrical vessel serves just to keep the floating stick approximately vertical since the sides of the vessel do not let the stick take a horizontal position ("lie flat").

13. The volume of the hollow V_{hol} and the volumes of the stopper V_{st} and glass V_{gl} enter into the obvious equality

$$V_{hol} = V_{st} - V_{gl}$$

Let the mass of the stopper be m_1. If the stopper is being weighed in water (the so-called hydrostatic weighing) the balance will show a smaller mass m_2 since in accordance with the Archimedes law the stopper is subjected to the action of the buoyant force. It is easy to see that the apparent decrease of the stopper mass $m_1 - m_2$ is equal to the mass of the displaced water. Therefore, dividing this difference by the density of water ρ_1 we find the volume of the

displaced water and consequently that of the stopper:

$$V_{st} = (m_1 - m_2)/\rho_1$$

To determine the volume of the glass we have only to divide the mass of the stopper by the value of the density of glass ρ_2 taking it from the pertaining tables:

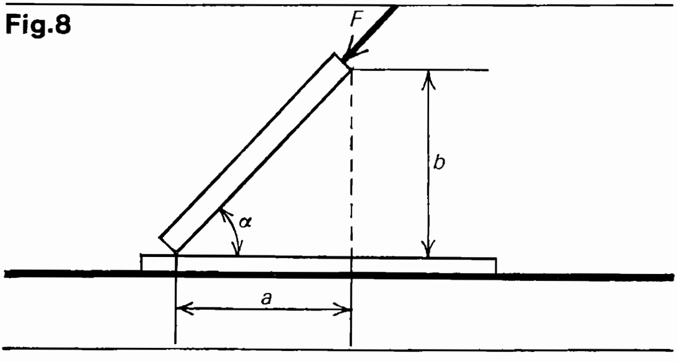
$$V_{gl} = \frac{m_1}{\rho_2}$$

Finally we obtain

$$V_{hol} = \frac{m_1 - m_2}{\rho_1} - \frac{m_1}{\rho_2}$$

14. Holding the stick vertical press it against the steel sheet and then gradually incline the stick,

Fig.8



continuing to put pressure on its top end (see Fig. 8). At a certain angle of inclination α the stick will start slipping on the steel sheet. This will happen at the

moment when the horizontal component of the force F applied to the top of the stick in the direction of its axis becomes equal to the force of friction. The horizontal component of force F is $F \cos \alpha$ and the friction force can be expressed as follows:

$$F_{fr} = k (P + F \sin \alpha)$$

where k is the unknown coefficient of friction, P is the weight of the stick and $F \sin \alpha$ is the vertical component of force F; this component (together with force P) presses the stick to the sheet. Equating the forces we obtain:

$$k (P + F \sin \alpha) = F \cos \alpha$$

Hence

$$k = \frac{F \cos \alpha}{P + F \sin \alpha}$$

According to the conditions of the problem the stick has but a small weight and consequently the first term of the denominator may be neglected, the more so as the force applied by the experimenter to the top of the stick is limited by his power and the strength of the stick. Then

$$k = \frac{F \cos \alpha}{F \sin \alpha} = \cot \alpha = \frac{a}{b}$$

Thus, in order to determine the coefficient of friction it is sufficient to measure a and b, and this is easy to do using a ruler.

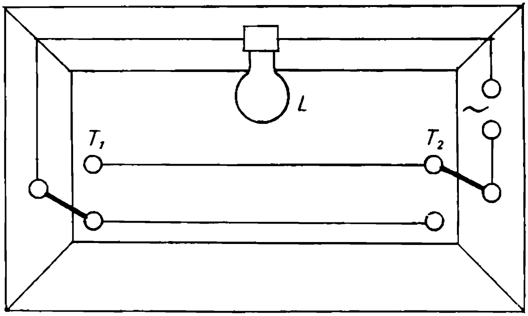
15. Moving the lenses away from the wall obtain a distinct image of the lamp filament. The lens which under this condition is nearer to the wall has the greater lens power.

16. Place the two lenses close to each other. If the system thus obtained will make the rays of light converge the focusing of the converging lens is stronger. Otherwise the diverging lens gives the stronger focusing.

17. Make use of two single-pole switches connecting them in the circuit supplying the current to the lamp as shown in Fig. 9.

18. Standing on the roof of the house let drop the can and simultaneously press the button of your

Fig. 9



stop-watch. Hearing the sound made by the can striking the ground stop the watch. The reading t of the stop-watch is the sum of the time taken by the falling of the can t_1 and the time t_2 necessary for the sound of the can striking the ground to reach the experimenter.

The time t_1 and the height of the house h enter into the following relationship:

$$h = \frac{gt_1^2}{2}$$

and the relationship for h and t_2 is of the form

$$h = ct_2$$

where c is the velocity of sound which we shall take in our calculation to be 340 m/s. Determining t_1 and t_2 and substituting the values obtained into the formula which comprises t_1, t_2 and t we obtain the irrational equation

$$\sqrt{\frac{2h}{g}} + \frac{h}{c} = t$$

which yields the height of the house.

In making an approximate calculation (especially if the house is of moderate height) the second term on the left-hand side may be considered small and so be neglected. In this case

$$\sqrt{\frac{2h}{g}} \approx t \quad \text{and} \quad h \approx \frac{gt^2}{2}$$

19. Measure the height h and diameter d_1 of the vessel with a vernier calliper and calculate the volume

$$V = \frac{\pi d_1^2}{4} \cdot h$$

Now, using a stop-watch measure the time t for the outflowing water to fill the jar. Then the quantity Q of water, which flows out per unit time, is

$$Q = \frac{V}{t} = \frac{\pi d_1^2}{4} \cdot \frac{h}{t}$$

On the other hand Q can be expressed as the product of the unknown outflow velocity v and the

area of the tap cross-section

$$Q = Sv = \frac{\pi d_2^2}{4} \cdot v$$

where d_2 is the tap diameter.

Equating the right-hand sides of the equalities, we obtain

$$v = \left(\frac{d_1}{d_2}\right)^2 \cdot \frac{h}{t}$$

Since d_2 can also be measured with the vernier calliper the problem is solved, in principle.

20. Point the hoze nozzle vertically upwards and determine, using a ruler, the height h to which the jet of water will rise. The outflow velocity v can now be found from the formula

$$v = \sqrt{2gh}$$

Multiplying the velocity obtained by the area S of the nozzle cross-section (its diameter d is measured with the ruler too), the flow Q of water, i.e. the amount per second which flows out, is determined:

$$Q = vS = \sqrt{2gh} \cdot \frac{\pi d^2}{4}$$

It is now possible to determine the time required to fill the tank, its capacity V being known:

$$t = \frac{V}{Q} = \frac{4V}{\pi d^2 \sqrt{2gh}} \approx \frac{0.9V}{d^2 \sqrt{gh}}$$

21. All bodies in air are acted upon by a buoyant force equal to the weight of the air displaced. Therefore, the results of the weighing of the ball pumped up to the pressure p can be represented as follows:

$$M = M_b + M_a - m$$

where M_b is the mass of the bladder and covering of the ball, M_a is the mass of air contained in the ball, and m is the mass of air displaced by the ball. It is easy to see that neglecting the volume of the bladder and covering (and consequently the Archimedes force which acts on them) the two last terms of the expression for the mass of the ball which contains air at atmospheric pressure are equal and the result of the weighing is the mass of the bladder and covering—M_b.

Since the volume V of the ball does not practically increase with pumping air into it the last term on the right-hand side remains practically constant as well as the first one, of course. Thus the difference $M - M_0$, where M and M_0 determined with the use of the balance are respectively the masses of the ball pumped up to the "working" pressure p and to atmospheric pressure p_0, will yield, after having been divided by the volume of the ball, the increase of air density from ρ_0 which corresponds to atmospheric pressure up to the value ρ at the pressure which is sought:

$$\frac{M - M_0}{V} = \rho - \rho_0$$

Since the volume of the ball does not change with pumping up, the density of the gas increases in direct proportion to the pressure:

$$\frac{\rho}{\rho_0} = \frac{p}{p_0}$$

Using the two last equalities we obtain the following expression for the value in question:

$$p = \frac{p_0}{\rho_0}\rho = \frac{p_0}{\rho_0}\left(\rho_0 + \frac{M - M_0}{V}\right) =$$
$$= p_0 + \frac{p}{\rho_0} \cdot \frac{M - M_0}{V}$$

The difference of the masses in this expression is determined with the use of a balance, the volume of the ball is calculated from the value of the diameter which is measured by a ruler (see problem No. 7), and the values of p_0 and ρ_0 are taken from tables (they are 1 atm and 1.293 kg/m^3, respectively). If a more precise result is wanted, one should take into account the change of air density with temperature, i.e. instead of the value 1.293, which corresponds to 0 °C, substitute in the last formula the value of density which corresponds to the air temperature at the moment.

22. First of all, taking care not to break the bulb, remove the holder and, submerging the bulb in water, find with a ruler the increase Δh_1 in the level of the water in the vessel. Then, without taking out the bulb, break off the tip (the part of the tube through which the lamp is evacuated and filled with inert gas). Some water will then enter into the bulb and the water level in the vessel will fall and be now by Δh_2 higher than it was initially. These data are sufficient for the solution of the problem.

Indeed, at the pressure p which is to be determined the gas occupied completely the volume V_1 of the lamp which, neglecting the volume of the glass walls of the bulb, can be written down as the product of the cross-sectional area S of the vessel and the increase in the water level in the vessel with the bulb submerged:

$$V_1 = S\Delta h_1$$

When the tip was broken the gas pressure in the bulb became equal to the atmospheric (the pressure of the water column above the bulb in comparison

with that of the atmosphere can be neglected) and the gas volume diminished to

$$V_2 = S\Delta h_2$$

Taking the temperature of the gas to remain constant we can write according to Boyle's law

$$pS\Delta h_1 = p_0 S\Delta h_2, \quad \text{hence} \quad p = p_0 \frac{\Delta h_2}{\Delta h_1}$$

The changes Δh_1 and Δh_2 in the water level are measured with a ruler and p_0 is read from the barometer or taken to be equal to 1 atmosphere (760 mm Hg).

23. First of all, as in the above variant of the solution of the problem, remove the holder, taking care not to break the bulb; then weigh the bulb. Let its mass be m_1. Now submerge the bulb in water and break off the tip. When the water ceases to enter into the lamp and the space free of water will be only that occupied by the gas compressed to atmospheric pressure p_0 take the bulb out and again determine its mass. Let the mass of the bulb partly filled with water be m_2. Now fill the bulb completely with water and determine its mass once more. Let it now be m_3.

It is easy to see that the difference $m_3 - m_1$ is that mass of water, which completely fills the bulb. Dividing the difference by the density ρ of water we find the internal volume of the lamp, in other words, the volume of gas at the unknown pressure p.

On the other hand, the difference $m_3 - m_2$ divided by the density of water is the volume of the same gas at the atmospheric pressure p_0.

Taking the temperature of the gas to remain unchanged when the gas is compressed we can

write, according to Boyle's law,

$$p\,\frac{m_3-m_1}{\rho}=p_0\,\frac{m_3-m_2}{\rho}$$

Hence the pressure sought is

$$p=p_0\,\frac{m_3-m_2}{m_3-m_1}$$

24. If the tube is vertical with the open end up then the air in the tube is at a pressure

$$p_1=p_0+\rho gh$$

where p_0 is the atmospheric pressure, ρ the density of mercury, g the acceleration of gravity and h the height of the column of mercury. This pressure compresses the air in the tube to the volume

$$V_1=l_1S$$

where l_1 is the length of the column of air and S is the area of the cross-section of the tube.

When the tube is vertical with the open end down the pressure is

$$p_2=p_0-\rho gh$$

and the volume of the air

$$V_2=l_2S$$

Taking the temperature to be the same in both cases we have, according to Boyle's law,

$$(p_0+\rho gh)\,l_1S=(p_0-\rho gh)\,l_2S$$

Hence

$$p_0=\rho gh\,\frac{l_2+l_1}{l_2-l_1}$$

The density of mercury and the acceleration of gravity are taken from tables and l_1, l_2 and h are measured with a ruler.

If it is sufficient to express the atmospheric pressure in mm of Hg the formula takes a simpler form:

$$p_0 = h\frac{l_2 + l_1}{l_2 - l_1}$$

25. Assume the pan to contain water that has been cooled down to 0 °C in a refrigerator (there are small pieces of ice floating in the water). We put the pan on the gas stove and note the time by the clock. Let the interval of time to the moment when the water starts boiling be τ_1 and that to the complete evaporation of the water τ_2.

If the burning gas yields q joules per second then the quantities Q_1 and Q_2 of heat, required to heat the water and to turn the heated water into steam, respectively, can be written down as follows:

$$Q_1 = mc(t_{100^\circ} - t_{0^\circ}) = 100mc = q\tau_1$$

$$Q_2 = mr = q\tau_2$$

where m is the mass of water poured into the pan, c the specific heat of water, and r the latent heat of steam. Dividing the terms of the first equality by the corresponding terms of the second one we obtain:

$$\frac{100c}{r} = \frac{\tau_1}{\tau_2}$$

Hence

$$r = 100c\frac{\tau_2}{\tau_1}$$

Since an accurate determination of the heat losses (heat radiated to the ambient air, heat

required to heat the pan) is not possible, the result obtained cannot be claimed to be very precise.

26. Connect the battery, the coil of wire and the ammeter in series, and connect the voltmeter so that it shows the voltage across the coil of wire. Write down the readings of the instruments and calculate the resistance of the coil of wire at room temperature:

$$R_t = \frac{U}{I}$$

Then fetch some snow from outside, bury the coil in the snow and having waited a little till the snow starts melting and the wire has acquired the temperature of the snow determine in the same way the resistance of the coil R_0 at the temperature of melting snow, i.e. at 0 °C. Using now the relationship between the resistance of the wire and its temperature

$$R_t = R_0 (1 + \alpha t)$$

find the temperature of the air in the room

$$t = \frac{R_t - R_0}{R_0 \alpha}$$

In calculating t use the value of the thermal coefficient of resistivity of copper taken from the reference book. In the range of room temperatures for pure copper $\alpha = 0.0043$ deg^{-1}. If the impurities content in the copper of which the wire was manufactured is moderate and the electric measuring instruments have a precision grade of 0.1* then the temperature of

* Very good instruments! The instruments usually employed in engineering are of 0.5, 1.0 and even 2.0 grades.

the air can be determined with an error of considerably less than 1 degree.

27. By the method described in the solution of the preceding problem determine the resistance of the wire first at the temperature of melting ice (R_0), then at the temperature sought (R_t) and lastly in boiling water (R_{100}). Using now the system of equations

$$R_t = R_0(1+\alpha t); \quad R_{100} = R_0(1+100\alpha)$$

you can determine the thermal coefficient of resistivity

$$\alpha = \frac{R_{100}-R_0}{R_0 \cdot 100}$$

as well as the unknown temperature of the air in the room

$$t = \frac{R_t - R_0}{R_{100}-R_0} \cdot 100$$

28. Given an electric lamp and an energy meter mentioned in the conditions of the problem, the duration of the walk can be determined as follows.

Switch the lamp on at the moment the girl goes out and simultaneously write down the reading of the meter. The second reading is taken when the girl comes back. Knowing the quantity A of electric energy consumed it is possible to calculate the duration t of the walk by the expression

$$t = \frac{A}{N}$$

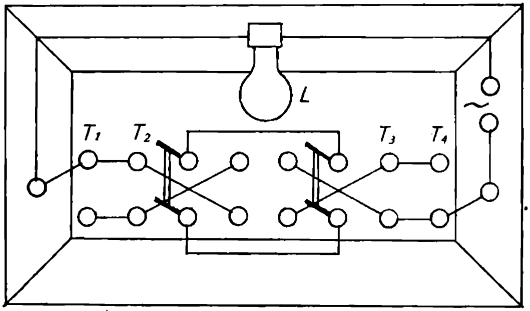
where N is the power of the electric lamp stamped on its bulb or bulb-holder.

In electric meters used in apartments the disc makes 1250 turns in registering a consumption

of energy of 1 kW-h. Thus with a lamp of 100 W the disc will make 125 turns in one hour. This can be read easily from the usual meter scale registering whole turns of the disc.

29. The simplest solution of the problem is obtained for the case when the lamp can be switched on

Fig.10



and off from four different stations with the circuit illustrated in Fig. 10.

It is easy to see that by adding a corresponding number of switches it is possible to arrange a ircuit which permits the appliance to be switched on and off from any number of different stations.

30. To make the little cube of mass m rotate with the disc in a circle of radius R at ν revolutions per second a centripetal force F_{cp} must be applied to the cube by the disc:

$$F_{cp}=\frac{mv^2}{R}=4\pi^2\nu^2mR$$

where v is the linear velocity of the c.g. of the cube.

This expression shows that with a constant angular velocity the centripetal force will increase monotonously with the increase in the radius of rotation. However, the value of the friction force F_{fr} which plays the part of the centripetal force cannot be higher than

$$F_{fr} = kmg$$

where k is the coefficient of friction and g is the acceleration of gravity.

Having measured with a ruler the limit value R_l, at which the little cube ceases to hold on the rotating disc and is thrown off, we find, equating the two above expressions,

$$k = 4\pi^2\nu^2 \frac{R_l}{g}$$

The value of ν in this expression is also known: it is equal to 33, 45 or 78 rpm.

31. Bind the small weight to the end of the thread and thus obtain a pendulum whose length is equal to the height of the room. Since the mass of the thread is very small you may take the pendulum to be "a simple" one, i.e. you may use the formula relating its period of oscillation T to its length l and the acceleration of gravity g and having the following form:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Having determined, using a watch, the period of oscillation of the pendulum (you need only count the number of oscillations of the pendulum in a sufficiently long lapse of time and then divide

the latter value by the former), calculate by the above formula the length of the thread l, i.e. the height of the room, taking from the handbook the value of g corresponding to the geographic latitude you are at or simply taking it to be 9.8 m/s^2. Determine the length and width of the room in the same way, and then by multiplication, the volume.

If the length of the pendulum proves too great (the room is large), this making inconvenient the determination of its period of oscillation, you can determine half the dimension in question by folding the thread in two.

32. The simplest approach seems to be as follows. Gradually shortening the thread of the pendulum which consists of a thread and a ball obtain the coincidence of the periods of the pendulum and the metronome. Having measured then the length of the thread and the radius R of the ball it is possible to calculate from the formula

$$T = 2\pi\sqrt{(l + R)/g}$$

the period of the pendulum which will be equal to that of the metronome. This procedure should be repeated for the greatest possible number of positions of the small weight on the scale of the metronome.

The changing of the length of the thread can be dispensed with. In this case the period of the pendulum is calculated only once. Then for each position of the small weight of the metronome one determines the number of complete oscillations of the pendulum corresponding to a certain number of clicks of the metronome. Multiplying the number of oscillations of the pendulum by its period the working time of the metronome is determined. Dividing this time value by

the number of clicks of the metronome its period is obtained. For a more detailed discussion of this method see the solution of problem No. 75.

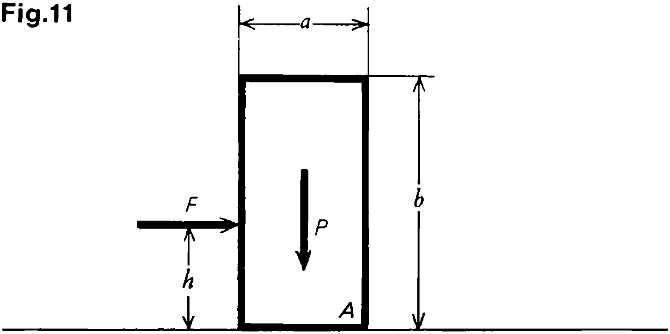
33. Place a ladder against the wall of a high building and make a mark at a distance of 4.9 m from the base line. From this height a stone will fall in time t:

$$t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\cdot 4.9\ \text{m}}{9.8\ \text{m/s}^2}}=1\ \text{s}$$

It remains to find a position of the small weight on the scale such that during the falling of the stone the metronome performs one oscillation.

34. If a horizontal force F (see Fig. 11) is applied to the bar at a short distance h from its base

Fig.11



and F is greater than the maximum force of friction $F_{fr}=kP$ the bar will start moving. However, if the force F is applied at a suffcient height the bar may topple over without moving from its place. This will happen

if the moment of the force F about an axis passing, for example, at right angles to the plane of the drawing through point A happens to be greater than the moment of the gravity force P about the same axis:

$$Fh > P \cdot \frac{a}{2}$$

where a is the width of the bar.

It is necessary to find that point of application of force F at which the transition from the first case to the second is observed. Then the coefficient of friction can be found from the system of equations

$$F = kP; \quad F \cdot h = P \cdot \frac{a}{2}$$

Solving it we obtain

$$k = \frac{a}{2h}$$

This shows that the experimental determination of the coefficient of friction is possible only if the height of the bar b satisfies the following condition:

$$b > \frac{a}{2k}$$

35. If the two spheres are placed side by side on an inclined plane and let free, the copper sphere rolling down will lag behind the aluminium one. The cause of this phenomenon is the following: in a rotational motion (and the rolling down can be considered as the combination of a translational and a rotational movement) the acceleration is determined not by the mass of the body but by its moment of inertia which is greater for the copper sphere, since its particles are at a greater mean distance from the axis of rotation. (You can read about the moment of inertia any textbook of physics for colleges.)

36. Support the lath at its midpoint (for instance, balancing it on the back of a chair) and fasten the body to be weighed to one of its ends. A wire loop is thrown on the other arm of this lever. By pulling the wire down vertically the system can be easily brought into equilibrium. The nearer the loop to the point of support the greater is the force necessary to obtain equilibrium. With this distance small enough the force can become greater than the tensile strength of the wire and the latter will break. Using the condition of equilibrium for the lever we can write that at the moment when the wire breaks

$$Pl_1 = \frac{\pi d^2}{4} \sigma l_2$$

where P is the unknown weight, l_1 the distance between the body being weighed and the point of support of the lever, d the diameter of the wire, σ the ultimate tensile strength of copper and l_2 the distance between the wire loop and the point of support.

The distances l_1 and l_2 are read from the marks of divisions on the lath and σ is taken from the handbook.

In order to find the diameter of the wire use the method whose principle is clearly seen by referring to Fig. 7, the lath being again used as a graduated ruler.

37. Place the mirror in a horizontal position and put the ball on it. If the ball is placed at a point other than the lowest it will start moving on the mirror surface. Clearly, if the ball moves without rotating (i.e. if it slides over the surface of the mirror) its movement is completely similar to that of a pendulum whose length is $R - r$ (Fig. 12). Then by the for-

mula for the pendulum

$$T = 2\pi\sqrt{(R - r)/g}$$

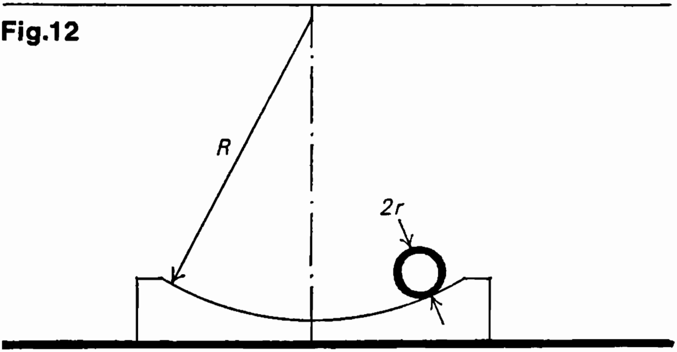
we can find the sought value

$$R = \frac{gT^2}{4\pi^2} + r$$

The period T is determined using a stop-watch and r is given.

Since the friction is usually great enough to make rotate the ball which is moving on the surface

Fig.12



of the mirror the above result does not agree with the experimental one. The real result is

$$T = 2\pi\sqrt{\frac{1.4\,(R-r)}{g}} \quad \text{and} \quad R = \frac{gT^2}{5.6\pi^2} + r$$

38. They should proceed as shown in Fig. 13. First the grown-up should cross the river walking on the planks and take the place of the child; then the child will walk over the planks.

39. Assume that the lightning flashed between points A and B of two neighbouring clouds (see

Fig.13

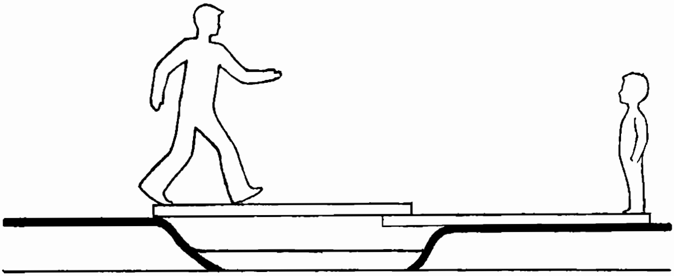
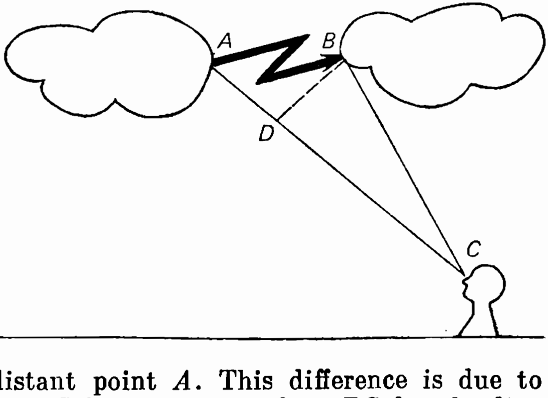


Fig. 14). A man standing at point C will first hear the sound coming from point B, then the sound from

Fig.14



the more distant point A. This difference is due to the distance AC being greater than BC by the line

segment AD. If the observer is placed in a favourable position (i.e. points A, B and C are practically in one straight line) then the difference between AD and AB is but small. Therefore calculating the distance AD as a product of the duration of thunder and the sound velocity one can take it to be equal to the length of the lightning.

40. If the experimenter is close to the bell he will hear the sound at the moment the blow is struck. However, with an increase in the distance from the bell the blows the experimenter observes visually and those he hears cease to coincide since it takes considerably more time for the sound waves to cover the distance from the bell to the man than for the travel of light (in most practically important cases the light can be taken to propagate instantaneously). At first this difference in time will increase but then it will start decreasing and at a certain distance the man will again see the blow struck and hear it simultaneously. As the distance from the bell increases this phenomenon will take place periodically.

The first coincidence of light and sound signals will occur at a distance which the sound covers during the interval between two blows. At this moment the man seeing the striking of the blow will hear the sound of the preceding blow. Since in the formulation of the problem it is stated that the interval between blows equals one second it is sufficient to measure the distance from the bell to the point where the visual and the sound signals again start coinciding and thus find the numerical value of the sound velocity.

41. The solution of the problem is shown in Fig. 15. Place the ruler in a vertical position and mark the length of the shadow B_1C_1 on the ground,

then measure it and the length BC of the shadow thrown by the tree.

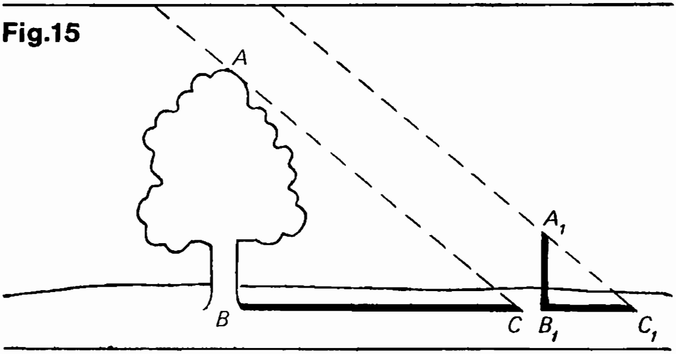
From the similarity of the triangles ABC and $A_1B_1C_1$ we have

$$AB = BC \cdot \frac{A_1B_1}{B_1C_1}$$

where A_1B_1 is the known length of the ruler.

42. Press the button of the stop-watch at the moment when the value v_1 of the speed on the signal

Fig.15



screen is superseded by the next one, v_2, and stop the watch when v_2 is in its turn superseded by v_3.

If the car starts moving at a speed v_1 just before the first change of signals on the screen, then it reaches the next traffic lights when the signal is green, having covered the distance S in an interval of time

$$t_1 = \frac{S}{v_1}$$

If the car begins to move at the moment of the changing of the signals on the screen from v_2 to v_3

(i.e. at the moment when the watch is stopped), then the speed of the car must be not less than v_2 and the distance S will be covered in an interval of time

$$t_2 = \frac{S}{v_2}$$

Obviously the difference $t_1 - t_2$ is equal to the reading of the stop-watch τ. Thus

$$\tau = \frac{S}{v_1} - \frac{S}{v_2}$$

Hence

$$S = \frac{v_1 v_2}{v_2 - v_1}\tau$$

For example, if $v_1 = 45$ km/h, $v_2 = 50$ km/h and $\tau = 8$ s, then

$$S = \frac{45\ \text{km/h} \cdot 50\ \text{km/h}}{50\ \text{km/h} - 45\ \text{km/h}} \cdot \frac{8}{3600}\ \text{h} = 1\ \text{km}$$

43. The boys should measure the distances S_1 and S_2 traversed by them from the moment they pushed off each other to full stop. Multiplying these distances by the corresponding masses, the coefficient of friction k and the acceleration of gravity g they can find amounts of work consumed by the braking forces

$$A_1 = km_1gS_1 \text{ and } A_2 = km_2gS_2$$

which of course are equal to the initial kinetic energies

$$A_1 = \frac{m_1v_1^2}{2} \text{ and } A_2 = \frac{m_2v_2^2}{2}$$

Hence we have the following two equalities:

$$kgS_1 = \frac{v_1^2}{2} \text{ and } kgS_2 = \frac{v_2^2}{2}$$

Dividing the terms of the first equality by the corresponding terms of the second one we obtain

$$\frac{S_1}{S_2}=\left(\frac{v_1}{v_2}\right)^2$$

On the other hand, the momenta acquired by the boys at pushing off must be equal, i.e.

$$m_1v_1=m_2v_2 \quad \text{or} \quad \left(\frac{v_1}{v_2}\right)^2=\left(\frac{m_2}{m_1}\right)^2$$

Equating the ratios of the velocities squared we find for the quantity sought

$$\frac{m_2}{m_1}=\sqrt{\frac{S_1}{S_2}}$$

Since the distances S_1 and S_2 have been measured the problem is solved. It should be noted however that a reliable result can be obtained only if the measurements have been performed several times and the mean result is taken into account.

44. Place the lath perpendicular to the ground surface and measure with the tape-line the length l_1 of the shadow thrown by the lath. Then recede a little from the bank, again place the lath in a vertical position and again measure the length l_2 of its shadow. From the similarity of the triangles ABC_1 and $A_1B_1C_1$ (see Fig. 16), the corresponding sides being in the same ratio, we obtain the following proportion:

$$\frac{H}{h}=\frac{S_1}{l_1}$$

(the designations are shown in the drawing). Similarly, from the triangles ABC_2 and $A_2B_2C_2$ we have

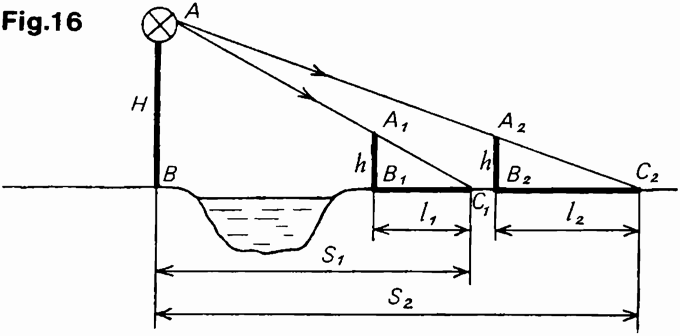
$$\frac{H}{h}=\frac{S_2}{l_2}$$

From the two proportions we find S_1 and S_2:

$$S_1=\frac{H}{h}l_1 \quad (1); \quad S_2=\frac{H}{h}l_2 \quad (2)$$

Subtracting the first expression from the second we

Fig.16



obtain

$$S_2-S_1=\frac{H}{h}(l_2-l_1)$$

Hence we obtain the height H of the post:

$$H=\frac{S_2-S_1}{l_2-l_1}\cdot h$$

In order to answer the second question of the problem divide equation (1) by (2):

$$\frac{S_1}{S_2}=\frac{l_1}{l_2}$$

Using the properties of derived proportions we obtain

$$\frac{S_1}{S_2-S_1}=\frac{l_1}{l_2-l_1}$$

Hence the distance S_1 to the post:

$$S_1=\frac{S_2-S_1}{l_2-l_1}l_1$$

The values h, l_1, l_2 and S_2-S_1 required to calculate H and S_1 are measured with the tape-line.

45. The following solution is possible, for instance.

You can place the pistol in a horizontal position and measure the distance l which the bullet will cover. If the elevation of the barrel with respect to the ground is h, then the bullet will be in flight during the time t

$$t=\sqrt{2h/g}$$

and cover a horizontal distance l:

$$l=vt$$

Eliminating the time t we obtain from these two equations

$$v=l\sqrt{\frac{g}{2h}}$$

46. Pointing the pistol barrel vertically upwards determine, using a stop-watch, the time t_0 from the moment the shot is fired to the moment the bullet touches the ground.

The height h to which a body thrown vertically upwards with an initial velocity v_0 will rise after a lapse of time t can be determined from the equation

$$h=v_0t-\frac{gt^2}{2}$$

Putting in this expression $h = 0$, which corresponds to the start as well as to the end of the flight of the bullet, we obtain a pure quadratic equation

$$\frac{gt^2}{2} - v_0 t = 0$$

which has two solutions of the form

$$t_1 = 0 \text{ and } t_2 = \frac{2v_0}{g}$$

The first, as is easily seen, corresponds to the moment of firing the shot, the second to the moment of falling of the bullet to the ground. Therefore, putting t_2 to be equal to t_0, the complete time of flight of the bullet, we obtain $t_0 = 2v_0/g$. Hence

$$v_0 = \frac{gt_0}{2}$$

A simpler solution may be suggested. Since the velocity of the bullet at the upper end of its trajectory is zero we can obtain from the expression

$$v_t = v_0 - gt = 0$$

the time of ascent $t = \frac{v_0}{g}$. Then, we have

$$t_0 = \frac{2v_0}{g}$$

since the ascent and descent take the same time.

However, firstly, the last proposition requires substantiation (and the proof is not so simple) and, secondly, it is useful to familiarize oneself with the above method of reasoning, which can be appropriate in solving more complex problems.

47. Let the boy and the girl throw the ball horizontally, one after the other. Measure the distances

to which the ball has been thrown, S_1 and S_2, in the two cases, using a tape-line. The length of the flight of the ball can be written down as the product of the initial velocity and the time of flight. The latter value is found from considering the vertical motion. If the elevation at which the ball starts flying is h then

$$t=\sqrt{\frac{2h}{g}}$$

and for the distances flown we obtain $S_1 = v_1 \sqrt{2h_1/g}$ and $S_2 = v_2 \sqrt{2h_2/g}$ respectively. Dividing the first expression by the second, term by term, we obtain for the ratio of the distances of flight

$$\frac{S_1}{S_2}=\frac{v_1}{v_2}\sqrt{\frac{h_1}{h_2}}$$

Hence the ratio of the velocities

$$\frac{v_1}{v_2}=\frac{S_1}{S_2}\sqrt{\frac{h_2}{h_1}}$$

Thus in solving the problem it is sufficient to measure with a tape-line the distances the ball has been thrown to and the elevations from which the ball started flying. To make the measurements and the solution simpler one can equalize h_1 and h_2 asking the shorter of the competing persons to step on a pedestal of appropriate height. Then

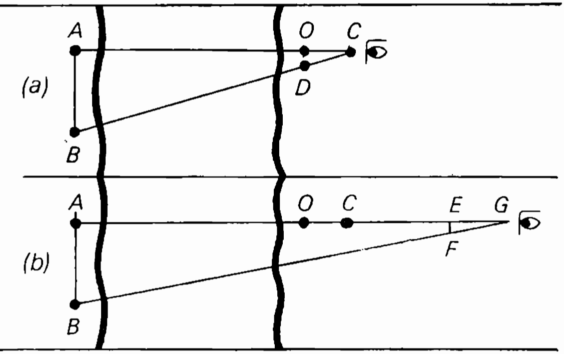
$$\frac{v_1}{v_2}=\frac{S_1}{S_2}$$

48. Standing close to the river select two distinctly visible objects on the opposite bank very near to the water and pick a blade of grass such that it bridges the interval between the two objects when

it is held at arm's length. Of course, you must keep one eye shut.

Now fold the blade in two and recede from the bank until the blade again bridges the interval between the two objects. Then measure the distance

Fig.17



between the two points at which you had stood. This distance will equal the width of the river.

Indeed, from the similarity of the triangles ABC and ODC (see Fig. 17a) we have the proportion

$$\frac{AB}{OD}=\frac{AC}{OC} \tag{1}$$

where AB is the distance between the objects selected, AC the distance to them from the first station of observation, OD the dimension of the blade of grass and OC the length of your stretched arm.

In the same way from the similarity of the triangles ABG and EFG (see Fig. 17b) we have

$$\frac{AB}{EF}=\frac{AG}{EG}$$

and, since $EF = OD/2$ and $EG = OC$

$$\frac{2AB}{OD} = \frac{AG}{OC} \tag{2}$$

Dividing term by term equality (2) by (1) we obtain

$$2 = \frac{AG}{AC}$$

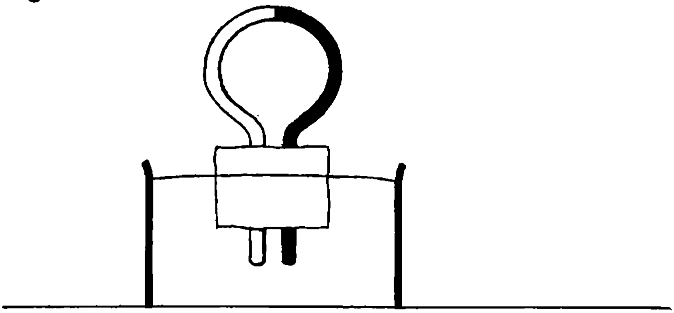
Hence

$$AC = \frac{AG}{2} = CG$$

The distance CG is measured in paces.

49. It is easy to construct a galvanic cell from the set of objects listed in the conditions of the problem using a solution of sal ammoniac in water as

Fig.18

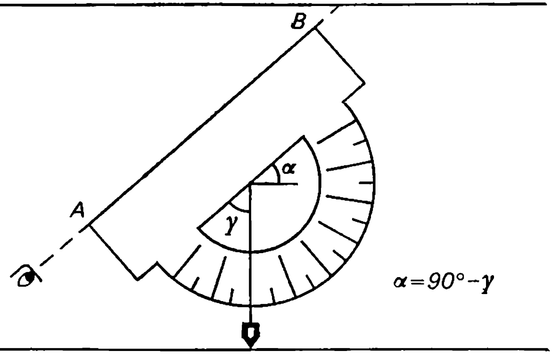


an electrolyte and copper wire and zinc for electrodes. "Floating electrodes" can be made by piercing a cork with the wire, as shown in Fig. 18.

If the electrodes are then connected to a solenoid consisting of a few turns of wire a current

will flow in the circuit and the solenoid will take the direction of the magnetic meridian. Since the signs of the poles of the element are known (copper is the positive pole, zinc the negative one) it is easy, using for instance the right-hand screw rule, to determine

Fig.19



what the poles of the solenoid magnet are and then in what directions the Earth's north and south poles lie.

50. First of all construct from the protractor and the small weight a primitive clinometer, an instrument for measuring the angle α between the horizontal and the direction AB to a certain point (the instrument is shown in a diagrammatic form in Fig. 19). Now put the saucer with mercury on the ground and recede to a distance from which you can see the reflection of the top of the tower, then measure the angles α and β with the clinometer shown in Fig. 20. From the triangle ABC we have

$$AB = BC \cot \alpha = (L - h) \cot \alpha$$

On the other hand

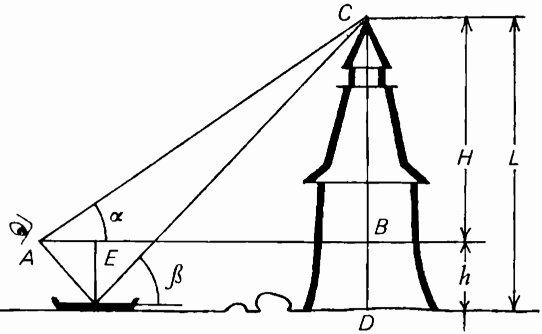
$$AB = AE + EB$$

and

$$AE = OE \cot \beta = h \cot \beta \text{ and}$$
$$EB = DC \cot \beta = L \cot \beta$$

Substituting these expressions in the preceding formula and equating the two expressions for AB, we

Fig. 20



obtain

$$(L - h) \cot \alpha = h \cot \beta + L \cot \beta$$

Hence

$$L = \frac{\cot \alpha + \cot \beta}{\cot \alpha - \cot \beta} \cdot h = \frac{\tan \beta + \tan \alpha}{\tan \beta - \tan \alpha} \cdot h$$

where h, the distance from the ground surface to the man's eyes, is known from the conditions of the problem.

51. The atmospheric pressure diminishes with increasing elevation above sea level. On the other

hand, with a decrease in pressure the temperature of boiling water becomes lower too. This fact is sometimes made use of by mountain-climbers in determining the elevation attained. Precise results are obtained using special tables, but for a rough calculation it is sufficient to remember that the lowering of the boiling point of water is approximately 0.3 °C for every 100 m of elevation.

52. Let the intensity of illumination measured by a luxmeter at a distance R (unknown) from the source of light be E_1. Recede by a distance d and again use the luxmeter. Let the new reading of the instrument be E_2. According to the laws of illumination we can write

$$E_1 = \frac{I}{R^2}; \quad E_2 = \frac{I}{(R+d)^2}$$

Solving the first equation for R and substituting the expression obtained into the second we obtain an irrational equation in I, the candle-power:

$$E_2 = \frac{I}{\left(\sqrt{\frac{I}{E_1}} + d\right)^2}$$

The value d necessary for the calculation is determined with a tape-line.

53. A body at rest with respect to a rotating system, for example, a fly sitting on the disc of a record-player, is *in this system* acted upon by a force, called the centrifugal force of inertia, which tends to move the body away from the axis of rotation. If the body starts moving with respect to the rotating system (the fly begins to crawl) another force independent of the direction of movement appears; this force, called

the Coriolis force, acts at right angles to the velocity of the moving body.

People moving over the surface of the Earth do not perceive this force owing to its being small, the rotative motion of the Earth being comparatively slow. However, it is just the Coriolis force which makes all the rivers flowing in many different directions in the northern hemisphere undermine more the right banks, and in the southern hemisphere the left ones (this law was formulated by the Russian Academician K.M. Behr in 1857). The same cause makes the winds and sea currents in the northern hemisphere deviate to the right and in the southern one in the opposite direction.

The Coriolis force vanishes only in case the velocity of the moving object is directed parallel to the axis of rotation of the system. On the contrary, if the movement is perpendicular to this direction the Coriolis force is maximum.

Using the above data we can solve the problem as follows.

Standing on the rotating platform so as to face its periphery let the ball roll away from yourself. Owing to the rotation of the platform and to the Coriolis force caused by this motion the trajectory of the ball will not be a straight line. If the ball deviates to the right the rotation is counterclockwise (looking at the platform from above), otherwise it is clockwise.

Note that the problem can be solved without resorting to the Coriolis force using the first law of dynamics (the law of inertia).

54. Particles of soap lather that fall on pure water move swiftly away from one another: this is

due to the lowering of the forces of surface tension with the dissolving of soap in water.

55. If the boys move closer by pulling at the rope the accelerations of their boats will be equal only in case the masses of the boats are equal, since the forces acting on the boats are equal according to Newton's third law:

$$a_1 = \frac{F}{m_1} \text{ and } a_2 = \frac{F}{m_2}$$

Then the distances covered by the boats till they meet must also be equal since the time of their movement is of course the same:

$$S_1 = \frac{a_1 t^2}{2} \text{ and } S_2 = \frac{a_2 t^2}{2}$$

Thus the boys may be sure that the masses of their boats are equal if they have achieved equality of the distances covered. It is easy to compare these distances, measuring equal stretches with the rope.

56. Assume the man is standing in the prow of the boat which is not moving. Then the sum of their momenta is zero. Neglecting the resistance of water (which is permissible at low speeds) this sum must remain unchanged in case the man begins to move towards the stern. Therefore we can write

$$m_1 v_1 + m_2 v_2 = 0$$

where the subindices 1 and 2 are used for the man and the boat, respectively. Multiplying both sides of the equation by t, the time in which the man covers the distance from prow to stern, we obtain

$$m_1 v_1 t + m_2 v_2 t = 0 \text{ or } m_1 S_1 + m_2 S_2 = 0$$

Hence

$$m_2 = -m_1 \cdot \frac{S_1}{S_2}$$

The minus sign represents the fact that the boat moves in the opposite direction to that of the man's movement and can therefore be neglected:

$$m_2 = m_1 \cdot \frac{S_1}{S_2}$$

In this expression S_1 and S_2 are the displacements of man and boat with respect to water, which does not move ("absolute" displacements). Taking into account that the man covers the distance l with respect to the boat the relationship between S_1 and S_2 can be written in the form

$$S_1 = l - S_2$$

Thus

$$m_2 = m_1 \frac{l - S_2}{S_2}$$

Consequently by measuring the length of the boat and the distance covered by it we can calculate the mass of the boat, since the man's mass is known.

Since in the last expression we have a ratio of the length $l - S_2$ to S_2 there is no necessity in expressing them in conventional units; we can, for example, break a twig and determine how many lengths of it go into the distances mentioned. It can be seen that we can dispense with the rope. However, it is expedient to cut two pieces off it equal to S_2 and $l - S_2$, and then to measure these two pieces.

57. It seems the tourist switched off his portable set at the moment he heard the first Moscow

(Greenwich) time signal and noted the time by his watch. He noted the time once more hearing the same Moscow (Greenwich) time signal transmitted by the loudspeaker at the camp. The difference of the two readings of time Δt multiplied by the sound velocity v gives the distance from the place of rest to the camp:

$$l = v \cdot \Delta t$$

Calculation shows that for a distance of the order of 3 km the relative error is not more than 10 per cent, which is not so bad after all.

It is easy to see that such a calculation is feasible not only during the transmission of the time signal: the last words of a sentence transmitted by radio may be used instead of the time signal.

However, using the Moscow time signal (it is transmitted every hour as an item of the "Mayak" programme) we can determine the approximate distance from the camp, if not too great, even without the watch. Indeed, let the fifth signal (remember that six signals are transmitted with an interval of one second between them) which the tourists hear on their set coincide with the first signal transmitted by the loudspeaker at the camp. This means that the time taken by the sound to travel from the camp to the place of rest is equal to 4 seconds. Multiplying this time by the sound velocity we find the distance from the camp, as the crow flies.

It is easy to see that the method used here resembles the one discussed in the solution of problem No. 40.

58. This problem is solved with the aid of Boyle's law. The skin-diver should dive to the bottom holding the graduate with the open end down and

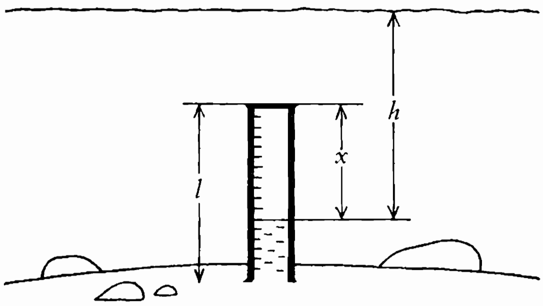
note to what division mark the level of the entering water will rise (see Fig. 21).

At normal atmospheric pressure p_0 the volume of air in the graduate was

$$V_1 = lS$$

where l is the height and S the area of the cross-section of the graduate. At the bottom of the lake the pressure

Fig. 21



will increase to

$$p = p_0 + \rho g h$$

where h is the depth of the lake which is to be determined, ρ the density of water and g the acceleration of gravity. Under the action of the pressure the volume of air in the graduate will diminish to

$$V_2 = xS$$

where x is the height of the air column in the graduate at the bottom of the lake.

Assuming the temperature and density of water at different depths to remain unchanged we have according to Boyle's law:

$$p_0 lS = (p_0 + \rho gh)\, xS$$

Hence after some simple transformations we have

$$h = \frac{p_0}{\rho g} \cdot \frac{l-x}{x}$$

It is quite irrelevant in what units the quantities l and x are expressed. In particular they may be substituted into the formula in terms of the scale units of the graduate. This is due to the fact that the final expression for h contains the *ratio* of uniform values of $l - x$ and x, and the ratio naturally is independent of the units in which the length has been measured. Since the atmospheric pressure changes comparatively slightly we can consider it to be constant and equal to 1.013×10^5 N/m^2. The density of water is also practically constant (1000 kg/m^3) and so is the acceleration of gravity (9.8 m/s^2).

A conical graduate may, obviously, be substituted for the cylindrical one. In this case Boyle's law is expressed by an equation of the form

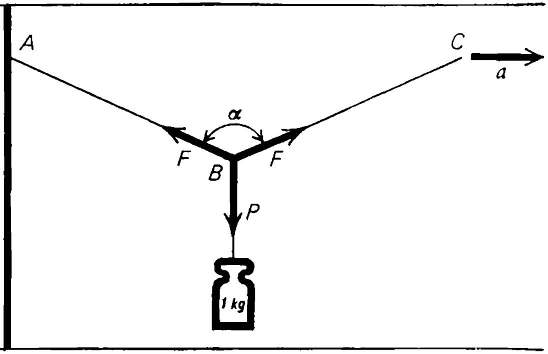
$$p_0 V_1 = (p_0 + \rho gh)\, V_2$$

where V_1 and V_2 are the volumes occupied by the air in the graduate at the surface and the bottom of the lake, respectively, which can be read from the scale.

It is desirable to have the marks in both cases reach to the top of the graduate. Otherwise immersing the graduate slightly into the water let some water enter it so as to make the level of water coincide with the initial point of the scale.

59. The weight should be suspended by the fishing-line as shown in Fig. 22 and the line pulled in the direction shown by the arrow a. The forces F

Fig. 22



and P are in the relation expressed in the form

$$P = 2F \cos\frac{\alpha}{2}$$

Hence

$$F = \frac{P}{2\cos\frac{\alpha}{2}} = \frac{mg}{2\cos\frac{\alpha}{2}}$$

Gradually tensioning the line make the angle α increase; the force of tension F increases too. Using the protractor, note the value of the angle α at which the line breaks; thus the permissible load can be calculated. If the line breaks even with $\alpha = 0°$ it should be folded in two and the procedure described repeated. Do not forget to divide the final result by two.

60. The formula

$$F=\frac{mg}{2\cos\frac{\alpha}{2}}$$

obtained in solving the preceding problem can be rewritten in the following form:

$$F=\frac{mg}{2\sqrt{1-\sin^2\frac{\alpha}{2}}}=\frac{mg}{2\sqrt{1-\left(\frac{AC:2}{AB}\right)^2}}$$

Tensioning the line determine the distance between the points A and C at the moment when the line breaks and also the half-length of the line, AB. These measurements are made with the tape-line.

61. Gradually increasing the frequency of rotation of the fishing-line with the small weight attached to it make the line break. The centripetal force acting on a rotating body

$$F_{cp}=\frac{mv^2}{R}=4\pi^2\nu^2\cdot mR$$

where ν is the number of revolutions per unit time (per second), m is the mass of the rotating body, R the radius of rotation, v the translational velocity of the body.

The expression shows clearly that the centripetal force must increase with the frequency of rotation. But it cannot increase beyond a certain limit, since this centripetal force is generated by the tension of the line and the latter cannot exceed

$$\frac{\pi d^2}{4}\cdot\sigma$$

where d is the diameter of the line and σ the ultimate tensile strength of the material from which the line was made.

Equating the last two expressions we obtain for the quantity in question the following formula:

$$\sigma = \frac{16\pi\nu^2 \cdot mR}{d^2}$$

The critical value of the frequency of rotation is determined by direct counting of the number of revolutions in a certain time at a velocity slightly less than the critical one.

62. First, using a stop-watch, we determine the time t_1 from the moment the stone falls into the water to the moment when the ripples produced by the falling stone reach the shore. We also determine the number n of the ripples reaching the shore per a certain time t_2. We can also approximately determine, using a ruler, the distance between two "crests" of the ripples or two "valleys", i.e. the length λ of the incoming ripples.

Obviously the distance we want to measure can be written down as follows:

$$x = vt_1$$

where v is the velocity of propagation of the ripples. Since

$$v = \lambda\nu$$

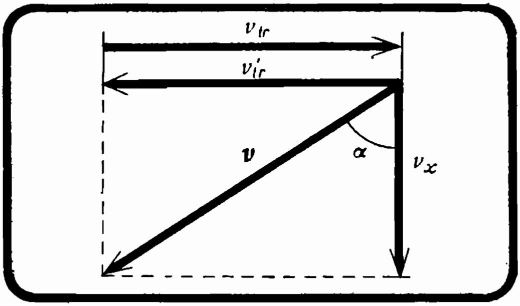
(ν is the frequency of oscillations of the incoming ripple; $\nu = n/t_2$) we have

$$x = \lambda\nu t_1 = \lambda \frac{t_1}{t_2} n$$

63. If the train is moving from left to right at a velocity v_{tr} the raindrops with respect to the

train have the same but oppositely directed velocity v'_{tr}. The raindrops also move downward with respect to the car at a velocity which we have to determine; we denote it by v_x. The vector of the resultant velocity

Fig. 23



v, as shown in Fig. 23, forms an angle α with the vertical, this angle satisfying the condition

$$\tan\alpha = \frac{v'_{tr}}{v_x}$$

From this expression we obtain the velocity of the falling raindrops in the form

$$v_x = v'_{tr}\cot\alpha$$

The angle α is read on the protractor and the velocity of the train v_{tr} is calculated from the time which it takes the train to cover the distance from one of the kilometre posts to the next (these posts are clearly visible even when it rains).

64. With respect to the motor-car the drop takes part in two motions simultaneously—vertical and horizontal. The resultant velocity v is directed

at a certain angle α to the vertical and (see the solution of the preceding problem)

$$\tan \alpha = \frac{v_0}{v_x}$$

where v_0 is the car speed and v_x the velocity of falling of the drop which we want to know. The tangent of the angle α can be found by measuring the sides of the right angle of the right-angled triangle formed by the trace of the drop (the hypotenuse) and the sides of the frame of the window (the other two sides of the triangle). The speed of the car can be read on the speedometer. After this we calculate v_x.

65. Tie the small weight to the thread and suspend this pendulum from the railcar ceiling. With uniform acceleration of the moving train the pendulum will be deflected until the resultant F of the gravity force P and the tension force R of the thread reaches a value sufficient to give the weight the same acceleration as that of the train.

From the similarity of the triangles AOB and ACD (see Fig. 24) we have

$$\frac{AD}{AC} = \frac{AB}{OB} \text{ or } \frac{ma}{mg} = \frac{x}{\sqrt{l^2 - x^2}}$$

Hence

$$a = g\frac{x}{\sqrt{l^2 - x^2}}$$

where l is the length of the pendulum and x its displacement from the position of equilibrium measured along the horizontal.

Thus, having measured the length of the pendulum and its displacement we are able to calculate the acceleration of the train.

66. From the triangle ACD (Fig. 24) we find:

$$(AD)^2 = (DC)^2 - (AC)^2 \text{ or } F = \sqrt{R^2 - P^2}$$

The last equality can be written in the following form:

$$ma = \sqrt{R^2 - P^2}$$

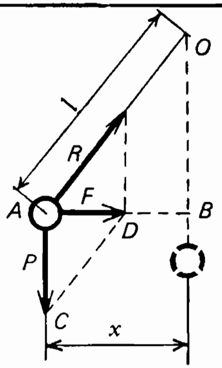
Hence

$$a = \frac{\sqrt{R^2 - P^2}}{m} = g\frac{\sqrt{R^2 - P^2}}{P} = $$

$$= g\sqrt{\left(\frac{R}{P}\right)^2 - 1}$$

where P is the reading of the dynamometer in the train when it is not moving or moving at constant

Fig. 24



speed, and R the reading when the train is uniformly accelerated.

67. Measuring the angle AOB with the protractor (see Fig. 24) we find from the relationship

$$\tan \angle AOB = \tan \angle ACD = \frac{F}{P} = \frac{ma}{mg} = \frac{a}{}$$

the acceleration of the train:

$$a = g \tan \alpha$$

68. With the changing of the temperature of the wheel from t_1 to t_2 its radius will change from $r_1 = r_0(1 + \alpha t_1)$ to $r_2 = r_0(1 + \alpha t_2)$ and the length of the circumference from $l_1 = 2\pi r_1 = 2\pi r_0(1 + \alpha t_1) = l_0(1 + \alpha t_1)$ to $l_2 = l_0(1 + \alpha t_2)$. Therefore the number of revolutions of the wheel on a stretch L will change from $N_1 = \frac{L}{l_1} = \frac{L}{l_0(1+\alpha t_1)}$ to $N_2 = \frac{L}{l_0(1+\alpha t_2)}$.

Dividing the first expression term by term by the second we obtain

$$\frac{N_1}{N_2} = \frac{1+\alpha t_1}{1+\alpha t_2}$$

Hence

$$\alpha = \frac{N_2 - N_1}{N_1 t_1 - N_2 t_2}$$

N_1 and N_2 are determined using a counter of revolutions and t_1 and t_2 with a thermometer.

For a steel wheel ($\alpha = 1.2 \cdot 10^{-5}$ deg^{-1}) with a radius of 0.5 m the number of revolutions on a covered distance of 100 km with an increase in temperature from -25 °C to $+25$ °C (this range makes it necessary to perform the experiment in different seasons—in summer and winter!) changes by but 19.

So it cannot be claimed that this method is a highly accurate one.

69. Having speeded up the car disengage the engine from the driving wheels and, noting on the speedometer the speed v, determine, using the same instrument, the distance S which the car will cover before it stops. Equating the kinetic energy of the car to the work done by the forces of resistance

$$\frac{mv^2}{2} = mg\,kS$$

we obtain the coefficient of resistance to the movement of the car:

$$k = \frac{v^2}{2gS}$$

The digit in the right small window of the speedometer (hundreds of metres) makes it possible to read the distance covered to an accuracy of 50 m (with some practice even of 30 m). The deflection of the speedometer pointer permits the speed to be read to an accuracy of about 2 km/h. Using the formulas of the theory of errors we can find that at a speed of 80 km/h and a distance covered of 600 m the relative error in determining k will be

$$\frac{\Delta k}{k} = \frac{2\Delta v}{v} + \frac{\Delta S}{S} = \frac{2 \times 2\ \text{km/h}}{80\ \text{km/h}} + \\ + \frac{30\ \text{m}}{600\ \text{m}} = 0.1 = 10\%$$

Not so bad a result.

70. In order to give the bar a uniform translational motion up the inclined plane a force F_{up} must be applied equal to the sum of the force of friction $F_{fr} = kP \cos \alpha$ and that component F_1 of the bar weight which is parallel to the inclined plane. Accord-

ing to Fig. 25 we have

$$F_{up} = kP\cos\alpha + P\sin\alpha$$

Similarly for the force F_{down} which gives the bar a uniform motion down the plane we have

$$F_{down} = kP\cos\alpha - P\sin\alpha$$

Subtracting the second equation from the first we obtain:

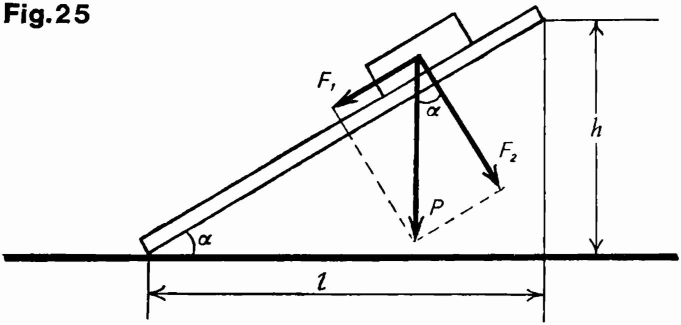
$$F_{up} - F_{down} = 2P\sin\alpha$$

hence

$$\sin\alpha = \frac{F_{up} - F_{down}}{2P}$$

The forces F_{up}, F_{down} and P can be determined with a dynamometer; using the last expression

Fig.25



it is possible now to find the sine of the angle of slope and then, with the aid of tables, the value of the angle.

If this angle is small we can determine it without having recourse to tables, since with small angles

$$\sin \alpha \approx \alpha$$

if, of course, the angle α is expressed in radians. In this case

$$\alpha \approx \frac{F_{up} - F_{down}}{2P}$$

Multiplying the value obtained by 57 degree/rad we obtain the approximate value of the angle of slope in degrees.

71. Using the leads make a circuit consisting of the lamp and the storage battery and place the mariner's compass beneath a straight part of one of the leads and near it. The magnetic field will make the pointer of the compass deflect. Knowing the direction of the deflection you can use the right-hand screw rule and determine the direction of the current in the circuit and thus the signs of the terminals of the storage battery.

The lamp is not obligatory because its purpose is only to limit the current in the circuit, and this is not necessary since the storage battery is switched on only for a short time.

72. Connect the leads to the terminals of the storage battery and dip the free ends into a glass of water. Electrolysis of the water will start in the glass; this can be observed by bubbles of gas appearing on the ends of the leads in water (in order to intensify the process you should add a drop of sulphuric acid to the water).

Since a molecule of water consists of two atoms of hydrogen and only one atom of oxygen and

with equal pressure equal volumes of gas must contain equal numbers of gas molecules, the amount of hydrogen produced by electrolysis must be twice that of oxygen. Therefore noting on which of the electrodes there are more bubbles you find where the hydrogen gas is evolved. It is now a simple thing to tell to what pole this lead is connected, since the hydrogen ions have a positive charge and this gas must be evolved on the cathode.

73. Connect the copper wires to the terminals of the storage battery and stick the free ends into a potato. The electric current flowing through it will produce electrolysis of the water in the potato. As a result of the process hydrogen will be evolved close to the lead connected to the negative pole of the storage battery, and oxygen near the lead connected to the positive pole. Interacting with copper oxygen forms oxides and hydroxides, whose ions colour the potato part near the corresponding lead light-blue-green. No colour will appear at the other lead.

Thus the lead near which the potato becomes green is connected to the positive pole of the storage battery.

74. Using a stop-watch determine the time t taken by the hoop, starting from rest, to roll down the motor road a distance l, measured by the speedometer of the car (this distance can also be calculated from the known length of the circumference of the wheel). In this time the potential energy of the hoop diminishes by

$$\Delta W = mgh = mgl \sin \alpha$$

where α is the angle we want to find. The loss of potential energy is equal to the kinetic energy acquired by

the hoop. Should the hoop *slide* on a plane its kinetic energy could be calculated by the formula

$$T = \frac{mv^2}{2}$$

where v is the velocity of the centre of the hoop, and m is its mass.

But when the hoop *rolls down* its kinetic energy consists of two terms expressing the kinetic energy connected with the translational motion and the kinetic energy of rotation:

$$T = T_{transl} + T_{rot}$$

Therefore to find the kinetic energy of the rolling hoop we must calculate the kinetic energy of the translational motion, neglecting the rotation, and then calculate the kinetic energy of rotation assuming the centre of the hoop to be at rest, and add the two values obtained.

The calculation of the energy of a rotating body is rather difficult, since points at different distances from the axis of rotation have different velocities. However, for a hoop the problem is very simple, since all its points are at equal distances from the axis passing through the centre of gravity; the distance is equal to the radius of the hoop.

If the centre of the hoop moves with respect to the ground at a velocity v, all the points of the hoop have this velocity with respect to the centre. Consequently the second term in the above expression can be written in the following form:

$$T_{rot} = \frac{m_1 v^2}{2} + \frac{m_2 v^2}{2} + \ldots + \frac{m_n v^2}{2} =$$

$$= \frac{v^2}{2}(m_1 + m_2 + \ldots + m_n) = \frac{mv^2}{2}$$

where m_i are masses of separate "points" of the hoop, whose sum is m—the mass of the whole hoop.

It follows that for the complete kinetic energy we obtain

$$T = \frac{mv^2}{2} + \frac{mv^2}{2} = mv^2$$

Taking the motion of the hoop to be uniformly accelerated we find the final velocity v when the hoop has covered a distance l, using the time t taken by the run:

$$v = \frac{2l}{t}$$

The expression for the kinetic energy of the hoop now takes the form

$$T = 4m\frac{l^2}{t^2}$$

Equating this value to the loss of potential energy we obtain

$$mgl \sin\alpha = 4m\frac{l^2}{t^2}$$

From this we can find the sine of the angle of slope of the motor road:

$$\sin\alpha = \frac{4l}{gt^2}$$

All the quantities in this expression are either known or can be directly measured.

It remains to say that the slope of the motor road being in most cases comparatively small, we can use the approximate equality

$$\sin\alpha \approx \alpha$$

(see the solution of problem No. 70).

75. Suspend the two pendulums side by side, so they can be observed together. Pulling aside the two pendulums, let them go simultaneously. At the initial moment the phases of the oscillations will be equal but gradually the pendulum which has a shorter period will "leave behind" the other one. However, after some time the oscillations will again be in phase.

Clearly if up to that moment the first pendulum has made n oscillations, the second will have made one oscillation less.

Therefore we can write

$$nT_1 = (n - 1)\ T_2$$

where T_1 and T_2 are the periods of the first and the second pendulum, respectively.

The expressio nobtained shows that knowing the period of one of the pendulums (given, see conditions of the problem) and the number n (counted in the experiment) we are able to find the period of the second pendulum:

$$T_2 = \frac{n}{n-1} T_1 \text{ or } T_1 = \frac{n-1}{n} T_2$$

76. Cut thin strips of all kinds of paper you have and immerse their ends into water. In the strip whose pores are smaller the water will rise to a greater height.

77. Touch the midpoint of one bar with the end of the other. If the former is a magnet the latter will not be attracted to it, since at the midpoint of a straight bar magnet there is as a rule a so-called neutral line. If, however, attraction is observed then the bar with which the first bar is touched is a magnet.

A straight bar, however, can be magnetized in such a way that one of the poles will be at the mid-point, the south, say, and two north poles will be at the ends of the bar (such a bar can be considered to consist of two magnets set to each other by their south poles). In this case one should draw the end of one bar along the other. If attraction is observed continuously then the former bar is a magnet (a pole will be necessarily at the end). If, however, the attraction is apparent only in some points when this bar is displaced along the other one, then it is the latter that is a magnet.

78. Put the two flasks in front of the table lamp and consider the path of the rays of the lamp through the two liquids. Since the index of refraction of water is $n_1 = 1.33$ and that of alcohol $n_2 = 1.36$ the rays of light, having penetrated the flask with alcohol, will converge at a point nearer to the flask than those which passed through the flask of water.

79. The simplest method is as follows. Using the stop-watch measure the time in which the carriage moved by the action of the load P placed on the scale of the installation covers a definite distance, for instance, from one end of the pedestal to the other. Then, taking the body under consideration off the carriage put weights on the scale in a quantity allowing the carriage to cover the same distance as in the first case. Then the total mass of the weights on the carriage equals the mass of the body.

80. An iron core with two coils on it can be considered as a transformer. If one of the coils is connected to a source of a.c. and the voltages V_1 and V_2 are measured on the two coils we can find the *ratio* of the numbers of turns of the coils from the following

proportion

$$\frac{V_1}{n_1}=\frac{V_2}{n_2}$$

However, these measurements do not make it possible to determine n_1 and n_2 *separately*. Therefore, let us wind over the two coils a third one with a known number of turns n. The above proportion can now be supplemented with a third ratio, containing the voltage V of the third supplementary coil:

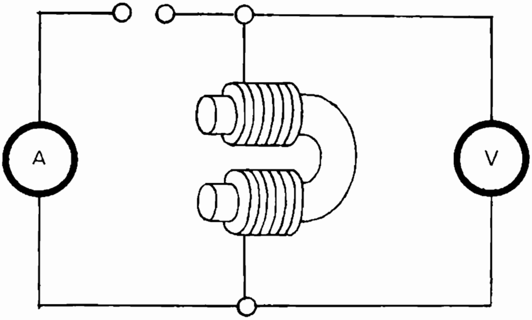
$$\frac{V_1}{n_1}=\frac{V_2}{n_2}=\frac{V}{n}$$

We have now

$$n_1=\frac{V_1}{V}n \text{ and } n_2=\frac{V_2}{V}n$$

The quantities V_1, V_2 and V are determined with a voltmeter and n by counting the number of turns in winding the supplementary coil.

Fig. 26



81. Make the circuit shown in Fig. 26. Now using Ohm's law determine V from the readings of the

voltmeter and I from the readings of the ammeter and then find the resistance R of the coil of the electromagnet:

$$R = \frac{V}{I}$$

After this measure with the micrometer the diameter d of the wire and calculate first its length l

$$l = \frac{RS}{\rho} = \frac{\pi d^2 V}{4\rho I}$$

and then its mass:

$$m = DlS = \frac{\pi^2 d^4 \cdot V \cdot D}{16\rho I}$$

In the last two formulas ρ is the resistivity of copper and D its density; these quantities can be taken from the handbook.

82. Let the mass of the empty capillary determined with a balance be m_1. Draw some mercury into the capillary by suction (of course, do it with a small rubber syringe, not with the mouth, as mercury is dangerous) and weigh the capillary once more. Let the mass be now m_2. It follows that the mass of the mercury column in the capillary is

$$m = m_2 - m_1$$

On the other hand, m can be expressed in terms of the length l of the column, its diameter d and the density of mercury D as follows:

$$m = D \frac{\pi d^2}{4} l$$

From the two above equalities we obtain:

$$d = \sqrt{\frac{4(m_2 - m_1)}{\pi D l}}$$

Since the length of the mercury column can be easily found with a ruler and the density of mercury can be taken from tables, the value d can be readily calculated.

83. Let the two cardboard discs be fixed on the motor axle at a distance l from each other so that their planes are at right angles to the axis. If a shot is fired from a rifle in a direction parallel to the axis when the motor is working then there will appear bullet-holes in both discs with an angle of shift between them

$$\varphi = \omega t = 2\pi n \frac{l}{v}$$

where ω is the angular velocity of rotation of the motor easily determined from the frequency of rotation n rev/s, t the time it takes the flying bullet to cover the distance l between the discs, and v is the sought velocity of the bullet.

From the above equality we obtain

$$v = \frac{2\pi nl}{\varphi}$$

The distance l is measured with a tape-line, the angle φ with a protractor and the frequency of rotation is known (given).

84. According to Hooke's law, with moderate stresses (i.e. in the domain of *elastic* strain) the elongation x of a spring is directly proportional to the applied force F:

$$F = kx$$

where k is the so-called coefficient of elasticity (or coefficient of stiffness), which depends on the material and dimensions of the spring.

Let the elongation of the spring fixed to a support be x_1 under the action of the known weight P_1 and x_2 under the action of the weight P_2 of the body. We have the two following equalities:

$$P_1 = kx_1, \quad P_2 = kx_2$$

Dividing the second equality by the first, term by term, we obtain:

$$\frac{P_2}{P_1} = \frac{x_2}{x_1}; \text{ hence } P_2 = \frac{x_2}{x_1} P_1$$

The elongations x_1 and x_2 are measured with the ruler and P_1 is known from the conditions of the problem.

85. Placing the bar on the plank lift the end of the latter until the bar starts moving. Fig. 25 (p. 94) shows that the force F_1 which tends to move the bar in the direction parallel to the inclined plane is $P \sin \alpha$. The force of friction between the bar and the plane can be written in the following form:

$$F_{fr} = kP \cos \alpha$$

where P is the weight of the bar and k the coefficient of friction which we want to find.

In uniform motion the force which produces the sliding of the bar is equal to the braking force, i.e.

$$P \sin \alpha = kP \cos \alpha$$

Hence we have the coefficient of friction in the form

$$k = \tan \alpha = \frac{h}{l}$$

Thus to find the coefficient of friction we have only to measure h and l.

86. In solving problem No. 70 we found the expressions for the forces necessary to start the bar moving uniformly up and down the inclined plane. Recall these expressions:

$$F_{up} = kP\cos\alpha + P\sin\alpha,$$

$$F_{down} = kP\cos\alpha - P\sin\alpha$$

Subtracting the second equation from the first we obtain the expression which permits us to find the sine of the angle of slope of the plane with respect to the horizontal

$$\sin\alpha = \frac{F_{up} - F_{down}}{2P}$$

Adding together the same equations we can obtain the expression for calculating the cosine of the same angle:

$$\cos\alpha = \frac{F_{up} + F_{down}}{2kP}$$

Taking the second powers of the two above equalities and adding them together we obtain:

$$1 = \left(\frac{F_{up} - F_{down}}{2P}\right)^2 + \left(\frac{F_{up} + F_{down}}{2kP}\right)^2$$

Hence we have for the coefficient of friction

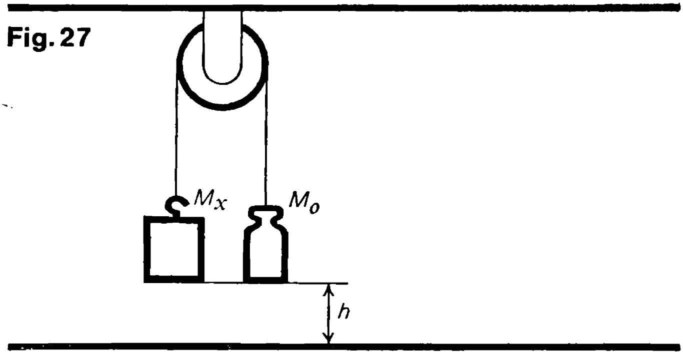
$$k = \frac{F_{up} + F_{down}}{\sqrt{4P^2 - (F_{up} - F_{down})^2}}$$

The forces F_{up}, F_{down} and P are determined by a dynamometer.

87. Fix the pulley at a certain height with respect to the floor (for instance, with screws, to the ceiling), and passing the cord over the pulley tie the weight and the body under consideration to the ends

of the cord. For the calculations made later on it is expedient to place initially these two objects at the same height as shown in Fig. 27. Then let the weight

Fig. 27



and the body move. Depending on the relation between the mass of the body M_x and that of the weight M_0 three cases are possible.

(a) The system is at rest. This will be the case if $M_x = M_0$. No further measurements are required, since the problem is solved.

(b) The weight moves down and the body up. In this case the mass of the weight must be greater than that of the body

$$M_x < M_0$$

Using the second law of dynamics we find the acceleration of the two objects:

$$a = \frac{M_0 g - M_x g}{M_0 + M_x} = \frac{M_0 - M_x}{M_0 + M_x} g$$

The distance covered in the time t by a body which is moving with a constant acceleration a with zero initial velocity can be found from the equation

$$h = \frac{at^2}{2}$$

Substituting in this equation the value of the acceleration a written above we obtain

$$h = \frac{1}{2} \cdot \frac{M_0 - M_x}{M_0 + M_x} gt^2$$

Hence we find that

$$M_x = \frac{gt^2 - 2h}{gt^2 + 2h} M_0$$

The value of M_0 is known from the conditions of the problem, g is a constant, and t is measured with the stop-watch. Thus, in order to solve the problem a method should be suggested of determining the distance h covered by the weight and body in time t (it is expedient to choose the distance to the floor as the one to be measured, see Fig. 27). The measurement can be performed by the method described earlier in the solution of problem No. 31 for determining the height, width and length of a room; the essence of the method is that a simple pendulum is made with a thread and a small weight, the length of the pendulum being made equal to the dimension to be measured (in our case this is the distance of the weight from the floor) and this dimension is calculated by the formula for the period T of the pendulum; T is determined using a stop-watch:

$$h = \frac{gT^2}{4\pi^2}$$

(c) If $M_x > M_0$ the expressions for the acceleration of the system and the mass of the body have the form

$$a = \frac{M_x - M_0}{M_x + M_0} g \text{ and } M_x = \frac{gt^2 + 2h}{gt^2 - 2h} M_0$$

All the quantities in the last expression are either known or can be found by methods discussed earlier.

It remains to add that in order to obtain a correct solution of the problem in all the three cases the friction of the pulley on its axle must be as low as possible and in the last two cases, i.e. (b) and (c), it is required that the mass of the pulley-block (to be precise, its moment of inertia) be as small as possible.

88. The bar must be heated to 100 °C in a can of water on a spirit-lamp. Then the bar is transferred into a calorimeter, into which a definite quantity of water, measured with a graduate, has been poured.

The initial and final temperatures of the water are measured with a thermometer and the mass m_1 of the bar is determined from the equation of thermal balance:

$$m_1c_1 (100° - t) = m_2c_2 (t - t_1) + \\ + m_3c_3 (t - t_1)$$

In this equation m_2 and m_3 are the masses of water poured into the calorimeter and of the calorimeter itself, respectively, c_1, c_2, c_3 are the specific heats of steel, water and the material from which the calorimeter is made, and t_1 and t the initial and final temperatures of water in the calorimeter.

89. The equation of thermal balance (see the solution of the previous problem) in this case takes

the form

$$m_1c_1(t_x - t) = m_2c_2(t - t_1) + \\ + m_3c_3(t - t_1)$$

where t_x is the temperature to be found.'

Thus the temperature can be readily determined by solving this equation. The mass of the bar m_1 is either considered known or is determined by the method discussed in problem No. 88.

90. Measure the current I_1 in the circuit, which comprises the storage battery, the ammeter and the known resistance R. This current

$$I_1 = \frac{\mathscr{E}}{R+r} \tag{1}$$

where $\mathscr{E}$ is the electromotive force (emf) of the storage battery and r its internal resistance. Then substitute the unknown resistance R_x for R in this circuit and again measure the current. The new value I_2 of the current can be expressed by the equation

$$I_2 = \frac{\mathscr{E}}{R_x+r} \tag{2}$$

Thus we have only two equations for determining three unknown values $\mathscr{E}$, r and R_x. It is well known that such a system has no unique solution and a third equation is required in order to find the unknowns. It is often found difficult to work out this equation. However, it can be done readily. It is sufficient to connect the known and the unknown resistances either in series or in parallel and to determine the current flowing in the new circuit. If the series connection is used we have

$$I_3 = \frac{\mathscr{E}}{R+R_x+r} \tag{3}$$

If the parallel connection is used

$$I_4=\frac{\mathscr{E}}{\frac{RR_x}{R+R_x}+r} \tag{4}$$

Complementing the equations (1) and (2) by (3) or (4) we obtain a system of three equations, which makes it possible to find the three unknowns, $\mathscr{E}$, r and R_x. In particular, using equations (1), (2) and (3) we have

$$R_x=\frac{I_2(I_3-I_1)}{I_1(I_3-I_2)}R$$

Composing the system of equations (1), (2) and (4) we have

$$R_x=\sqrt{\frac{I_1(I_4-I_2)}{I_2(I_4-I_1)}}R$$

91. In order to calculate the mean speed of the cyclist it is necessary to know the distance covered and the time. The value of the distance can be determined using the ruler and the map (the scale of the map is usually shown on it). A Grenet cell makes it possible to determine the time of travel.

When a galvanic cell is working the material of which the negative electrode is made is being dissolved in the electrolyte. In the Grenet cell, for example, the following reaction takes place:

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$

The mass m of zinc which passed into the solution can be found from Faraday's first law, knowing the time t during which the cell has been working and the current I flowing in the circuit which comprises the

cell:

$$m = kIt$$

where k is the electrochemical equivalent of zinc.

This shows us the method of solving the problem. Using connecting leads make a circuit which comprises the cell, a rheostat and an ammeter; the zinc electrode must be weighed beforehand. The circuit is closed when the cyclist starts and opened at the moment he comes back. The zinc is now weighed again and knowing the difference of the masses $m = m_1 - m_2$, the current I and the value of the electrochemical equivalent taken from tables the time of travel is determined.

92. It should be kept in mind that in an a-c circuit a capacitor behaves as a resistance whose value is inversely proportional to the capacity:

$$R = \frac{1}{\omega C}$$

(ω—circular frequency of the a.c.). Thus, the total resistance of the set of capacitors in the a-c circuit is proportional to $\frac{1}{C_{tot}}$. If the resistance of a set of equal resistances connected in the same way as the capacitors is R_{tot} then we can write the proportion

$$\frac{1}{C} : \frac{1}{C_{tot}} = R : R_{tot}$$

Hence we have

$$C_{tot} = \frac{RC}{R_{tot}}$$

The capacity of one capacitor is given in the conditions of the problem and R and R_{tot} can be

found from Ohm's law by making a circuit which consists of the storage battery, resistances, ammeter and voltmeter.

The same conclusion can be reached in a somewhat different manner. When resistors are connected in series their resistances are to be added and when capacitors are connected in series one must add the values inverse to their capacities. On the contrary when resistors are connected in parallel we must add the values inverse to their resistances, and when capacitors are connected in parallel we add their capacitances. It follows that in any combination of parallel or series connections the quantity $1/C$ behaves like the quantity R and this gives the same equality as above:

$$\frac{1}{C} : \frac{1}{C_{tot}} = R : R_{tot}$$

93. The resistance R of a piece of wire whose length l is equal to the height of the room can be determined from Ohm's law having made a circuit in which the storage battery is the source of current and the external part of the circuit (the load) is the piece of wire. If the ammeter connected in series with the piece of wire reads I and the voltmeter connected in parallel shows a potential difference V then

$$R = \frac{V}{I} = \rho \frac{l}{S}$$

where S is the cross-sectional area of the wire and ρ the resistivity of copper.

On the other hand, the mass m of the piece of wire found by weighing it with a balance can be expressed in terms of its length l, cross-sectional

area S, and the density D of copper as follows:

$$m = DlS$$

Multiplying the two last equalities term by term we obtain

$$\frac{mV}{I} = \rho D l^2$$

Hence

$$l = \sqrt{\frac{mV}{\rho D I}}$$

The quantities I, V and m are measured experimentally and ρ and D are taken from the handbook. The length and width of the room are found in the same way; the volume of the room can then be calculated.

If the voltage drop in the piece of wire equa- to the length of the room is small and hard to be detere mined with the voltmeter then another piece of wirl should be used in the circuit whose length is severa times (whole number) that of the room (lay the wire along the room several times).

94. It follows from the solution of the preceding problem that by weighing a piece of wire whose length is equal to the height of the room and using the value of the density of copper taken from a handbook, it is possible to calculate the product of the length of the wire and its cross-sectional area:

$$lS = \frac{m}{D}$$

Gradually increasing the mass of the load suspended by the wire we determine the critical load P_{cr} at which the wire breaks. The value of the

critical load is directly proportional to the cross-sectional area of the wire:

$$P_{cr} = m_{cr} \cdot g = \sigma S$$

where σ is the ultimate tensile strength of copper (see problems Nos. 36 and 61) whose value can be taken from the handbook.

Multiplying the last two expressions term by term we obtain after cancelling S

$$m_{cr} \cdot gl = \frac{m\sigma}{D}$$

Hence

$$l = \frac{m}{m_{cr} \cdot Dg}$$

The length and width of the room are then determined in the same way and after that the volume.

Unfortunately the ultimate strength of copper varies in rather wide limits with different kinds of copper. Therefore, good results can be obtained only in case the ultimate strength of the copper used is known to a sufficient accuracy.

95. The frequency of beats between the tuning fork and the organ pipe is N:

$$N = \nu_t - \nu$$

where ν_t is the frequency of the sound emitted by the tuning fork and ν is the frequency of the sound emitted by the organ pipe. The former value is constant (given) while ν changes with higher or lower temperature of air in the premises.

Indeed, the length λ of the sound wave coming from the organ pipe can be expressed in terms

of v, the velocity of sound propagation, and its frequency ν as follows:

$$\lambda = \frac{v}{\nu}$$

On the other hand, for an open organ pipe the length of the wave of the fundamental tone is

$$\lambda = 4l$$

where l is the length of the pipe. From the last two equalities we obtain:

$$\nu = \frac{v}{4l} \qquad (*)$$

Sound waves propagate in air at a velocity of the same order as that which the gas molecules have at the given temperature. Since the kinetic energy of the molecules is directly proportional to the Kelvin temperature T,

$$\frac{mv^2}{2} \sim T,$$

the velocity of the molecules and consequently that of sound is proportional to the square root of T:

$$v \sim \sqrt{T}$$

Hence the ratio of the velocity of sound v at temperature T to the value of this velocity v_0 at $T_0 = 0$ °C is as follows:

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}}$$

Using the equality (*) which represents the relationship between the frequency of the sound emitted by the organ pipe and the velocity of propagation of sound waves, we can rewrite the last expression in

the following form:

$$\frac{\nu}{\nu_0}=\sqrt{\frac{T}{T_0}}$$

where ν_0 is the frequency of the organ pipe at 0 °C which is equal to 440 Hz according to the conditions of the problem.

Substituting the obtained value of the frequency of the organ pipe at temperature T into the initial expression for the frequency of beats we obtain

$$N=\nu_t-\nu_0\sqrt{\frac{T}{T_0}}$$

Hence for the temperature T in the laboratory we have the expression

$$T=T_0\left(\frac{\nu_t-N}{\nu_0}\right)^2=T_0\left(\frac{\nu_t-N}{\nu_t}\right)^2$$

This expression is readily checked. Substituting in it $N = 0$ we find $T = T_0$. Actually at $T = 0$ °C the frequencies of the tuning fork and the organ pipe coincide and there are no beats.

96. Break the hack-saw blade (better a discarded one) in two. If the blade had been magnetized the two halves will interact.

97. First determine the mass m of the whole plate. Then using an angle-ruler draw a rectangle with known sides, cut it out with scissors and determine the mass m_0 of this rectangle using a balance. If the plate is of uniform thickness its mass m is to the mass m_0 as the area S of the whole plate is to the area S_0 of the rectangle:

$$\frac{m}{m_0}=\frac{S}{S_0}$$

Hence

$$S = S_0 \cdot \frac{m}{m_0}$$

The area S_0 is easily determined with the L-square.

If it is undesirable to cut the plate outline its contour with a pencil on a sheet of cardboard or strong paper, cut out the figure obtained and perform all the above measurements on this figure.

98. If the density of the substance decreases in solidifying, the piece of solid material thrown into the melt will float on the surface. This is how ice behaves in water since the density of ice is 1.1 times less than that of water. On the contrary, a piece of solid iron submerges in molten iron; this means that the density of iron increases in solidifying.

In the first case the volume of the substance increases when the melt solidifies, in the second the volume diminishes.

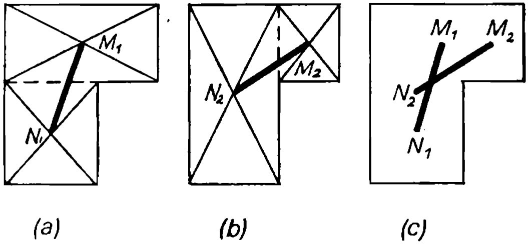
99. Let us mentally divide the plate into rectangles as shown by a dotted line in Fig. 28*a*. The centre of gravity of one rectangle is at point M_1, at which its diagonals intersect. Similarly we find the c.g. N_1 of the other rectangle. It follows that the c.g. of the plate must be in the straight line M_1N_1.

By a similar reasoning we find that the sought point must be in the straight line M_2N_2, where M_2 and N_2 are the centres of gravity of the rectangles drawn in Fig. 28*b*. Now, if the c.g. is simultaneously in two straight lines it must coincide with their point of intersection (see Fig. 28*c*).

Try now, applying the method worked out above, to find the centres of gravity of the figures shown

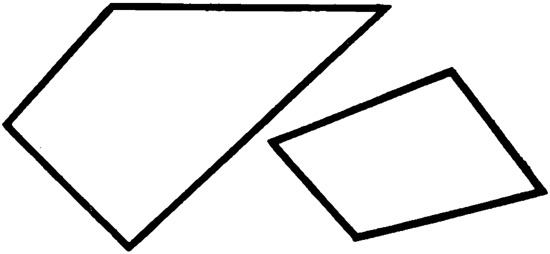
in Fig. 29 and both having the shape of irregular quadrilaterals.

Fig. 28



100. In order to solve this problem recall the left-hand rule, which is used to determine the direc-

Fig. 29

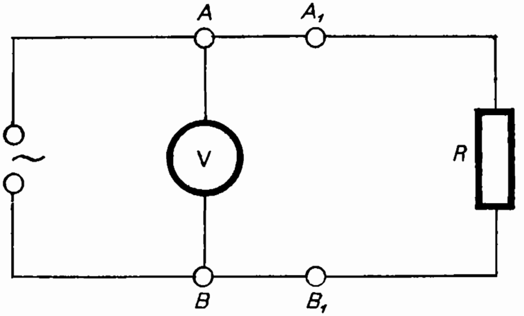


tion of the force acting on a current carrying conductor in a magnetic field.

If the lamp is supplied with a.c. then the magnet held close to it will make the lamp filament vibrate and the outline of the filament will become blurred. If it is a d.c. the filament will be distinctly visible since it will only be deflected sideways from its initial position.

101. You should open the circuit which supplies current to the lamp of the torch and connect the two coils to the free ends, first one coil then the other.

Fig.30



The resistance of the current coil is very small and its inclusion in the circuit does not practically change the incandescence of the lamp. On the contrary, the resistance of the voltage coil is high and when it is in the circuit the lamp will not burn at all.

102. Connect the voltmeter first to points A and B (see Fig. 30), then to points A_1 and B_1 somewhat to the right of the first pair. Let the readings of the voltmeter be V and V_1 respectively.

If $V > V_1$ the source of current is to the left of points A and B: the reading of the voltmeter

when transferred from points A and B to points A_1 and B_1 becomes lower owing to voltage drop in sections AA_1 and BB_1.

If $V < V_1$ the battery is at the right end of the circuit. In this case the reading of the voltmeter becomes higher owing to the voltage drop in the same sections.

The problem is solved in the same way in case of a-c and d-c circuits, only the corresponding type of voltmeter must be used. It is important to have a sufficiently sensitive voltmeter to record the rather small change in the potential difference when transferring the voltmeter from position AB to position A_1B_1.

103. Take one part from the first box, two parts from the second, three from the third one and so on to the last one from which ten parts are taken; then weigh all these parts together.

If the parts in all the boxes were made correctly their total weight would be P_1, say. Since in the nth box each part weighs 10 g less than required, the reading of the balance P_2 will be $10\,n$ g less than P_1:

$$P_1 - P_2 = 10\,n$$

Hence we find n, i.e. the number of the box containing rejected parts:

$$n = \frac{(P_1 - P_2)}{10}$$

104. If a periodically working lamp (flash-light) is used for lighting whose frequency of flashes is equal to the number of revolutions of the spindle, then the part will be visible all the time in the same position and its surface can be seen distinctly. The current supplied to the lamp should be from a generator with a smooth control of the frequency.

Instead of a flash-light and generator with variable frequency an ordinary bright lamp can be used, sending its light through a radial slit in a cardboard disc fixed on the axle of an electric motor whose speed can be varied.

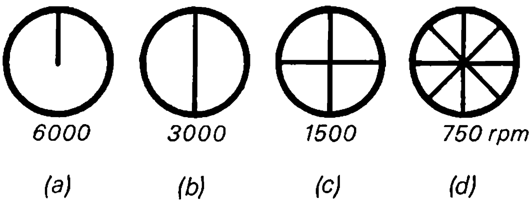
The effect which shows rotating bodies as if at rest owing to intermittent illumination is called stroboscopic. Some of its applications are also given in the next problem.

105. Draw a circle with compasses on cardboard and cut out the figure with scissors. On the cardboard disc thus obtained draw a radial line and fix the disc with glue on the motor axle so that the plane of the disc be at right angles to the axle. Connect the motor and a neon lamp to the mains. With an a-c supply the neon lamp will flash 100 times per second (6000 times per minute). If in the interval between two flashes the motor makes exactly one revolution (i.e. if the frequency of rotation is 6000 rpm) the radial line drawn on the disc will be seen all the time in the same position in spite of rotation, i.e. will seem to be at rest (Fig. 31*a*). If the frequency of rotation is halved then the shaft of the motor will make only half a revolution in the interval between two flashes and the line will be seen in a position on the same diameter with that in which it was seen at the preceding flash. As a result a longer line will be seen on a diameter across the disc (Fig. 31*b*). At a speed of 1500 rpm a cross will be seen on the disc (Fig. 31*c*), and at 750 rpm, two crosses will appear, one shifted 45° with respect to the other.

It should be noted that the picture shown in Fig. 31*a* will be seen not only when the disc turns one revolution in the interval between two flashes, but also when it makes two, three etc. revolutions in

this interval (i.e. at speeds of 12 000, 18 000 etc. rpm). The picture shown in Fig. 31*b* will be also seen at speeds of 9000, 15 000 etc. revolutions per minute (the disc has time to make 1.5, 2.5 etc. revolutions between the flashes). The cross (Fig. 31*c*) can be seen at speeds

Fig. 31



of 1500, 4500, 7500 etc. rpm and two crosses (Fig. 31*d*) at 750, 2250, 3750 etc. rpm. Having this in mind the motor speed should be increased gradually, approaching the desired speed from zero speed.

106. The astronaut should throw some object (if there is none his situation will be tragic) in the direction opposite to the rocket. Then, according to the law of conservation of momentum, which we applied in solving problem No. 56, the man will acquire a velocity V directed towards the rocket:

$$V = \frac{m}{M} v$$

where M and m are the masses of the man and the object, respectively, and v is the velocity of the latter.

This is how the rocket itself moves in space expelling the products of combustion of the fuel in a certain direction and moving because of this in the opposite direction.

107. The task can be performed by two methods. Let us discuss them one after the other.

(a) Assume that the body in question of mass m_1 has been placed on one pan of the balance, and a weight of mass m_2 on the other. Since the satellite and all the objects on it (including the body to be weighed and the weight) have the same acceleration in the gravitation field of the Earth (constantly "fall" on the Earth) the balance will be in a state of neutral equilibrium.

However, this equilibrium will be disturbed if the balance is set in uniformly accelerated motion with respect to the satellite. Indeed, to accelerate bodies of different mass with the same acceleration a different forces must be applied to them:

$$F_1 = m_1 a$$

and

$$F_2 = m_2 a$$

It is not difficult to achieve, at least approximately, zero deflection of the pointer of a balance with uniformly accelerated motion too, this requires the masses of body and weight to be equal.

(b) It is possible to use a spring balance as well—a dynamometer. If the body in question is suspended from the dynamometer and the system set in motion with a certain constant acceleration a the indicator of the instrument will register a certain force

$$F_1 = m_1 a$$

Then instead of the body a weight of known mass m_2 should be suspended from the dynamometer and the system set in motion with the same acceleration a. In this case the reading of the dynamometer will be

$$F_2 = m_2 a$$

Dividing one of the last two equalities by the other, term by term, we obtain

$$\frac{F_1}{F_2} = \frac{m_1}{m_2}$$

Hence

$$m_1 = m_2 \cdot \frac{F_1}{F_2}$$

When using a spring balance it is rather difficult to obtain equal accelerations.

Equality of accelerations can be realized by rotating the dynamometer at a constant angular velocity with first the body, then the weight suspended from the instrument. This obviously requires the use of a stop-watch. Better use, if possible, two dynamometers suspending the body in question from one of them and the weight of known mass from the other. Then holding the two dynamometers in one hand set them simultaneously in accelerated motion.

In solving this problem we make use of the equivalence of the accelerated motion of a system and the field of gravitation. The equivalence of the forces of gravitation and forces acting on bodies in systems which are in accelerated motion (inertia forces) served as one of the bases of the theory of gravitation, developed in Einstein's general theory of relativity (1915). Therefore, if the experimenter is on board a spaceship or a man-controlled satellite it is sufficient to start

the engines to make an "artificial force of gravity" appear and bring into action *any type* of balance.

108. If a spaceship is circling a planet with engines shut down, then the only force acting on the ship is F_{gr}—the force of attraction of the planet

$$F_{gr} = G\frac{Mm}{R^2}$$

where G is the gravitational constant, M the mass of the planet, m the mass of the ship and R its distance from the centre of the planet.

With a moderate altitude of flight the distance R can be taken equal to the radius of the planet (we assume it to be of an approximately spherical shape).

Let us express the mass of the planet in terms of its radius and mean density D

$$M = VD = \frac{4}{3}\pi R^3 D$$

Substituting this value of M in the preceding formula we have

$$F_{gr} = \frac{4}{3}\pi GRmD$$

Since the ship is moving on a circular orbit it is acted upon by a centripetal force which can be written in the following form:

$$F_{cp} = \frac{mv^2}{R} = m\omega^2 R = m\frac{4\pi^2}{T^2}R$$

where v is the linear and ω the angular velocity of the spaceship motion, T is its period of rotation round the planet.

Since it is the gravitation that plays the role of centripetal force here we can equate the right-

hand sides of the last two expressions and obtain the following expression for the density:

$$D = \frac{3\pi}{GT^2}$$

Thus having determined with the aid of a watch the period of rotation of the spaceship round the planet we can calculate its mean density.

Artificial satellites of the Earth placed on circular orbits have usually a period of rotation of about an hour and a half (5400 s). Substituting this value in the last formula we obtain the mean density of the Earth:

$$D = \frac{3 \times 3.14}{6.67 \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)} \times (5400\ \mathrm{s})^2} \approx$$
$$\approx 5000\ \mathrm{kg/m^3}$$

The automatic station "Luna-16" placed on its circular orbit round the Moon on October 17, 1970 had a period of 1 h 59 min (7140s); hence for the mean density of the Moon we obtain:

$$D = \frac{3 \times 3.14}{6.67 \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)} \times (7140\ \mathrm{s})^2} \approx$$
$$\approx 3000\ \mathrm{kg/m^3}$$

Actually, the densities of the Earth and the Moon are about 10 per cent higher. The difference is caused by the fact that the satellites usually fly at altitudes such that the difference between the radius of the planet and that of the orbit must be taken into account. If one substitutes in the above formula for density the values of T corresponding to orbits situated close to the surface of the Earth (this is practically impossible because of the resistance of air) or

the Moon which are 1 h 24 min and 1 h 48 min respectively, one obtains correct values of the densities 5500 kg/m^3 and 3360 kg/m^3.

For an orbit at a high altitude the formula for density has the following form:

$$D = \frac{3\pi}{GT^2}\left(1+\frac{h}{R}\right)^3$$

where h is the altitude of the orbit. Using this formula correct values of densities can be obtained in the case of high orbits.

109. Place the ruler in a vertical position on the background of black linen and illuminate it with an intermittent beam of lamplight through a slit in a uniformly rotating disc. Then open the shutter of a camera and drop a small ball from your hand so that it falls close to the ruler along it. The photograph obtained will show the ruler and a number of light spots which correspond to the positions of the falling ball at the moments when light impinged on it. The distances l_1, l_2, l_3 etc. between these positions can be easily read from the divisions on the ruler (Fig. 32).

The first light spot corresponds to the moment when the time t_0 elapsed from the initial moment of motion; in this lapse of time the ball covered a distance

$$S_0 = \frac{gt_0^2}{2}$$

The second time the picture of the ball was taken at the time t_1 elapsed from the initial moment of motion, when the distance covered was

$$S_1 = \frac{gt_1^2}{2}$$

The time t_2 and the distance S_2 correspond to the third position of the ball:

$$S_2 = \frac{gt_2^2}{2}$$

and so on. Obviously

$$l_1 = S_1 - S_0 = \frac{gt_1^2}{2} - \frac{gt_0^2}{2} = \frac{g}{2}(t_1^2 - t_0^2)$$

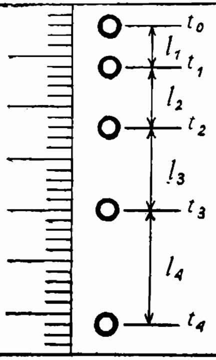
$$l_2 = S_2 - S_1 = \frac{g}{2}(t_2^2 - t_1^2)$$

Moreover

$$t_1 = t_0 + \tau \quad \text{and} \quad t_2 = t_1 + \tau = t_0 + 2\tau$$

where τ is the interval between two successive moments of the impinging of light on the ball equal to the period

Fig. 32



of rotation of the motor.

We can now write

$$l_1 = \frac{g}{2}[(t_0 + \tau)^2 - t_0^2] = \frac{g}{2}(2t_0\tau + \tau^2)$$

$$l_2 = \frac{g}{2}[(t_0 + 2\tau)^2 - (t_0 + \tau)^2] = \frac{g}{2}(2t_0\tau + 3\tau^2)$$

Subtracting the second equation from the first we obtain

$$l_2 - l_1 = g\tau^2$$

Hence

$$g = \frac{l_2 - l_1}{\tau^2}$$

The quantities l_1 and l_2 are read from the ruler and τ is determined from the known frequency of rotation of the motor.

110. It is sufficient to determine with the dynamometer the force P^* with which the planet attracts the weight (the weight of this weight on the planet). Then the acceleration of gravity g^* is found as follows:

$$g^* = \frac{P^*}{m}$$

where m is the mass of the weight given in the conditions of the problem.

111. Using the spring-balance the weight P^* (force of gravity) of the weight on the planet should be determined. According to the law of gravitation the force P^* is expressed in terms of the mass m of the weight, the gravitational constant G, the mass M of the planet and its radius R:

$$P^* = G \cdot \frac{Mm}{R^2}$$

Hence we have the following expression for the mass of the planet:

$$M = \frac{P^* R^2}{Gm}$$

G is a constant whose value can be taken from a reference book, P^* is known from the experiment and m and R are given.

112. Having made a simple pendulum from the thread and the small weight determine its period T using the stop-watch. The acceleration of gravity on the planet g^* can now be found from the pendulum formula:

$$g^* = \frac{4\pi^2 l}{T^2}$$

the length of the pendulum being known (conditions of the problem).

On the other hand, the acceleration of gravity can be expressed using the law of gravitation as follows:

$$g^* = \frac{P^*}{m} = G \cdot \frac{M}{R^2}$$

where M is the mass of the planet, R its radius and G the gravitational constant.

Equating the right-hand sides of the equations we obtain for the mass of the planet

$$M = \frac{4\pi^2 R^2 l}{GT^2}$$

The radius of the planet is readily expressed in terms of C, the length of the equator of the planet:

$$R = \frac{C}{2\pi}$$

We now have the following expression for the mean density D of the planet matter:

$$D = \frac{M}{V} = \frac{6\pi^2 l}{GT^2 C}$$

All the quantities in this expression are known and D can be calculated.

113. The length of the rope could be estimated by comparing it to the height of the astronaut who happened to be near; it proved about 1 m.

The period of oscillations determined using a watch was about 5 seconds. Now using the pendulum formula we have

$$T = 2\pi \sqrt{\frac{l}{g}}$$

and are able to find for the acceleration of gravity the value

$$g^* = \frac{4\pi^2 l}{T^2} = \frac{4 \cdot 3.14^2 \cdot 1}{5^2} \approx 1.6 \text{ m/s}^2$$

which is very close to that obtained by more precise methods.

114. Having made a coil of the wire connect its ends to the galvanometer. If the planet has a sufficiently strong magnetic field the galvanometer will register, when the coil is turned, impulses of an induction current, which is generated owing to changes in the magnetic flux through the plane of the coil. In this way not only the existence of a magnetic field can be determined in principle but even the value and the direction of the vector of magnetic induction.

The idea discussed above has been used as the basis of the design of certain types of magnetometers employed in practice.

115. Besides the law of conservation of momentum which was used in solving problems Nos. 56 and 106 there exists also the law of conservation of the *angular momentum*. According to this law the astronaut who rotates in his hands a disc or some other object must turn in the opposite sense (of course, if

there are no obstacles—other bodies). When the astronaut has turned by the angle required he must cease rotating the disc.

Now suppose the astronaut happens to have no object, what then? In this case according to the law of conservation of momentum he is not able to change the value and direction of his velocity (with respect to the rocket, for example) but the law of conservation of angular momentum does not exclude the possibility of changing the orientation of the astronaut. In order to turn, for instance, clockwise (looking on him from above) it is sufficient for him to perform the following cycle of motions: extend the right arm sideways, then press it to his bosom, lower it to hang along his body, again extend it sideways, etc.

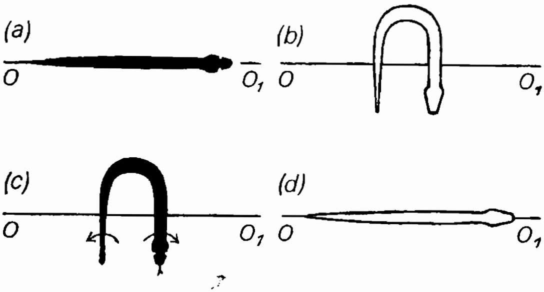
This problem resembles the famous problem of the falling cat. It is well known that the cat possesses the remarkable ability to land on its paws even in case it began to fall back downwards. Delayed (accelerated, to be exact) filming reveals that at the moment of falling the cat begins to swirl rapidly its tail. As a result of this the body of the cat turns in the opposite sense. The swirling of the tail stops when the paws look downwards.

The following example illustrates convincingly the possibility for a living creature to make a turn without outside help.

Let a snake stretch itself in space along OO_1 back up (Fig. 33*a*). The laws of nature and the make of the snake's body do not prohibit it to take the position shown in Fig. 33*b*. After this, turning the forward and hind parts of its body in the senses shown by arrows it can turn belly up (Fig. 33*c*) and then stretch itself along the line OO_1 (Fig. 33*d*). As a result of this

sequence of motions the snake will have turned about its axis 180°. (The example is an artificial one, for snakes usually move on the ground and in turning make use of the forces of friction.) However, in the air it could act as described above.

Fig. 33



116. The inhabitants of Venus could guess that their planet rotates and determine the direction of rotation in several ways.

The behaviour of the pendulum, for example, could be studied. In 1851 the pendulum was used by the French physicist Jean Bernard Léon Foucault to prove that the Earth rotates.

The pendulums intended for this are massive weights suspended by wires several scores of metres long (the longest pendulum in the world, 98 m long, was suspended in the St. Isaac cathedral, in Leningrad; Foucault's own pendulum placed in the Panthéon in Paris was 67 m long). The pendulum is fixed to the support by means of a universal joint that permits the mass to swing about a horizontal axis

in any vertical plane. Owing to the rotation of the Earth the plane in which the pendulum oscillates turns slowly with respect to fixed marks on the ground.

The forces of attraction by the Earth and tension of the wire which act on the mass of the pendulum lie in the plane of oscillations and cannot make the mass turn. For an observer outside the Earth the turning of the plane of oscillations is explained by the rotation of the surface of the Earth together with the fixed marks with respect to which the position of the plane is determined. At the Earth's pole, where the plane of oscillations remains in the same position which does not change with respect to the fixed stars, an observer on the surface of the Earth would see the plane of oscillation turn with an angular velocity equal to that of the rotating Earth but in the opposite direction.

At points situated between the poles the plane of oscillations of the pendulum (this plane passes through the vertical) cannot remain in a constant position with respect to the fixed stars and to some extent takes part in the rotation of the Earth. Accordingly, the plane of oscillations of the pendulum turns with respect to the fixed marks on the surface of the Earth, but slower than at the pole. At the equator this effect is not present.

The turning of the plane of oscillations of Foucault's pendulum can be explained from the viewpoint of an observer who is on the Earth. It is necessary in this case to take into consideration the Coriolis forces, which we met with in solving problem No. 53.

Another variant of solution of the problem under consideration demands the use of a gyroscope,

a massive top rotating at high speed and supported in universal (gimbal) joints.

If the centre of gravity of the gyroscope coincides with the centre of the suspension frame and the forces of friction are practically nonexistent (such a gyroscope is called balanced or free) the axis of the gyroscope will keep a constant direction with respect to the fixed stars. If the centre of gravity of the gyroscope does not coincide with the geometric centre of the suspension frame then the axis begins to describe a cone in space (this phenomenon is called precession). For a gyroscope whose mass is 1 kg, rotating at 30 000 rpm, the deviation of its centre of gravity by 1 μm produces a precession with an angular velocity of 1 deg/hour. Since the Earth rotates at a considerably greater angular velocity of 15 deg/h such a gyroscope makes it possible to reveal the fact that the Earth rotates which in fact was done for the first time by Foucault, in 1852. Of course, the slower a planet rotates the more precisely must the gyroscope be balanced, and the weaker must be the friction in the suspension joints.

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